INC 122 : Three-Phase Circuits

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Learning Outcomes

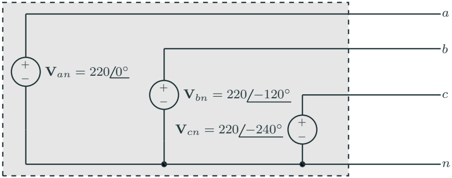
Students should be able to:

- Identify the properties of three-phase sources and the advantages of three-phase systems for power transfer
- Analyze a three-phase circuit with a balanced Wye or Delta load.

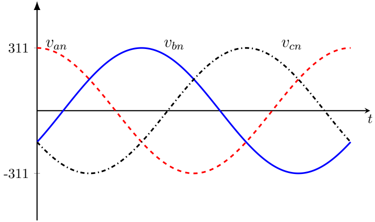
Balanced Three-Phase Circuits

- A power system supplied by one AC source is called a **a single-phase ($1 - \phi$) circuit.**
- At large power levels, the oscillatory behavior of the instantaneous power in a single-phase circuit puts severe pulsating strain on the generating and load equipment.
- **Polyphase circuits** with multiple sources were developed in response to this problem.
- The most common polyphase configuration is the **balanced three-phase ($3 - \phi$) circuit**, which has three AC sources arrange to achieve constant instantaneous power.
- This type of circuit delivers more watts per kilogram of conductor than an equivalent single-phase circuit. (4 vs 6 wires).
- For these reasons, almost all bulk electric power generation, distribution, and consumption take place via three-phase systems.

Balanced Three-Phase Circuits



(a)



(b)

$$V_{\text{peak}} = \sqrt{2}V_{\text{rms}}$$

$$\mathbf{V}_{an} = 220\underline{/0°} \text{ Vrms}, \quad v_{an}(t) = 220\sqrt{2}\cos(\omega t) \text{ V}$$

$$\mathbf{V}_{bn} = 220\underline{/-120°} \text{ Vrms}, \quad v_{bn}(t) = 220\sqrt{2}\cos(\omega t - 120°) \text{ V}$$

$$\mathbf{V}_{cn} = 220\underline{/-240°} = 220\underline{/120°} \text{ Vrms}, \quad v_{cn}(t) = 220\sqrt{2}\cos(\omega t + 120°) \text{ V}$$

Balanced Three-Phase Circuits

Let us examine the instantaneous power generated by a three-phase system. Assume that the voltage in the generator are

$$v_{an}(t) = V_m \cos(\omega t) \text{ V},\ v_{bn}(t) = V_m \cos(\omega t - 120^\circ) \text{ V},$$
$$v_{cn}(t) = V_m \cos(\omega t - 240^\circ) \text{ V}$$

If the load is balanced, the currents produced by the sources are

$$i_a(t) = I_m \cos(\omega t - \theta) \text{ A},\ i_b(t) = I_m \cos(\omega t - \theta - 120^\circ) \text{ A},$$
$$i_c(t) = I_m \cos(\omega t - \theta - 240^\circ) \text{ A}$$

The instantaneous power produced by the system is

$$\begin{aligned} p(t) &= p_a(t) + p_b(t) + p_c(t) \\ &= V_m I_m \left[\cos \omega t \cos(\omega t - \theta) + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \right. \\ &\quad \left. + \cos(\omega t - 240^\circ) \cos(\omega t - \theta - 240^\circ)\right] \end{aligned}$$

Using the trigonometric identity $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$

Balanced Three-Phase Circuits

We have

$$p(t) = \frac{1}{2}V_m I_m \left[\cos(\theta) + \cos(2\omega t - \theta) + \cos(\theta) + \cos(2\omega t - \theta - 240^\circ)\right.$$
$$\left. + \cos(\theta) + \cos(2\omega t - \theta - 480^\circ\right]$$
$$= \frac{1}{2}V_m I_m \left[3\cos(\theta) + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 120^\circ) + \cos(2\omega t - \theta + 120^\circ)\right]$$

We know that

$$\begin{aligned}\cos A + \cos(A - 120^\circ) + \cos(A + 120^\circ) &= \cos A \\ &\quad + \cos(A)\cos(120^\circ) + \sin(A)\sin(120^\circ) \\ &\quad + \cos(A)\cos(120^\circ) - \sin(A)\sin(120^\circ) \\ &= 0\end{aligned}$$

Then $$p(t) = \frac{3}{2}V_m I_m \cos(\theta) \text{ W}$$

The power delivery from a three-phase voltage source is very smooth (no sinusoidal terms).

Balanced Three-Phase Circuits

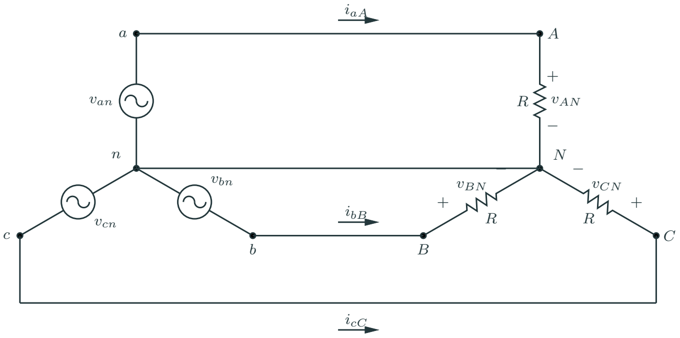
We are now in a position to make two statements of profound significance about this balanced three-phase circuit:

- The three-phase circuit requires fewer conductors than three single-phase circuits handling the same total power.
- The total instantaneous power is constant, rather than pulsating as in a single-phase circuit.

The latter property implies less vibration and mechanical strain at the generator and smoother power delivery to the load equipment.

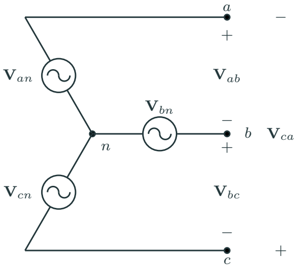
Balanced Three-Phase Circuits

Simple three-phase circuit.



- The reference terminals have been joined at node n, called the **neutral**.
- The remaining three source terminals and the neutral are connected by four wires to a load consisting of three equal resistances tied together at the load's neutral N.
- Such loads having three **identical** branches are said to be **balanced.**

Balanced Three-Phase Circuits



We turn to the frequency domain diagram:

- Descriptively named a wye (Y) generator.
- The neutral point n may or may not be exteranlly available, or safety considerations may call for it to be grounded.
- The generator is labeled with two sets of voltage phasors: the phase voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$, defined with respect to the neutral; $\mathbf{V}_{an} = V_\phi \underline{/\theta}$.
- line voltage $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$, defined across pairs of terminals.

The line volate voltages are related tothe phase voltages via $\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn}, \quad \mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn}, \quad \mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an}$. Consider only $\mathbf{V}_{ab}$, we have

$$\begin{aligned}\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} &= V_\phi \underline{/0^\circ} - V_\phi \underline{/-120^\circ} = V_\phi - V_\phi(\cos(-120^\circ) + j\sin(-120^\circ)) \\ &= V_\phi - V_\phi\left[-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right] = \sqrt{3}V_\phi\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right] = \sqrt{3}V_\phi \underline{/30^\circ}\end{aligned}$$

Balanced Three-Phase Circuits

- The **line voltages** are $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$. These voltages are easily measured and three phase generators are commonly rated in terms fo line voltage because phase voltages can be measured only when the neutral point is accessible.
- However, the phase voltages of a Wye generator correspond directly to the source voltages.
- From previous slide, we have

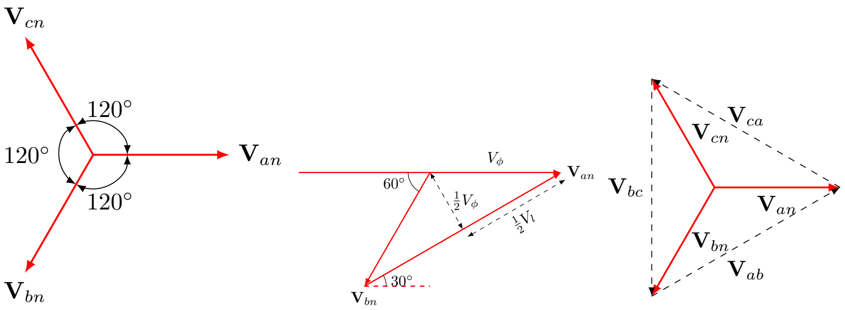
$$\begin{aligned}
&\mathbf{V}_{ab} = \sqrt{3}V_{\phi}\underline{/30^\circ}, \quad \mathbf{V}_{bc} = \sqrt{3}V_{\phi}\underline{/30^\circ - 120^\circ} = \sqrt{3}V_{\phi}\underline{/-90^\circ}, \\
&\mathbf{V}_{ca} = \sqrt{3}V_{\phi}\underline{/30^\circ + 120^\circ} = \sqrt{3}V_{\phi}\underline{/150^\circ}
\end{aligned}$$

- Hence $V_l = \sqrt{3}V_{\phi}$. (A line voltage is equal to $\sqrt{3}$ of a phase voltage.)
- If we rearranged $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$ head to tail, then they would form an equilateral triangle, confirming that

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

Balanced Three-Phase Circuits

The relationship between the phase and line voltages.



(a) voltage phasor

(b) $\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$

(c) Phase voltage and line voltage

Find $v_{ab}(t)$, $v_{bc}(t)$, and $v_{ca}(t)$ for a three-phase generator with $\mathbf{V}_{an} = 15\underline{/90^\circ}$ kVrms and $abc-$ sequence.

Solution: Since $V_\phi = 15$ kVrms, the rms line voltage is

$$V_l = \sqrt{3}(15\text{kV}) = 26 \text{ kVrms}$$

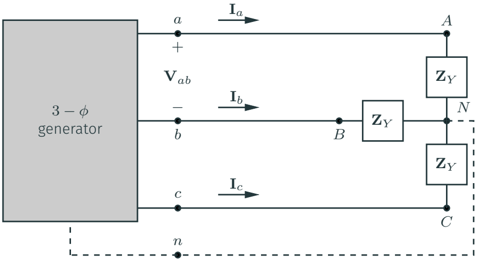
Then $\underline{/\mathbf{V}_{ab}} = \underline{/\mathbf{V}_{an}} + 30^\circ$, $\underline{/\mathbf{V}_{bc}} = \underline{/\mathbf{V}_{ab}} - 120^\circ$, and $\underline{/\mathbf{V}_{ac}} = \underline{/\mathbf{V}_{ab}} + 120^\circ$ $= \underline{/\mathbf{V}_{ab}} - 240^\circ$, so

$$\mathbf{V}_{ab} = 26\underline{/120^\circ} \text{ kV}, \qquad \mathbf{V}_{bc} = 26\underline{/0^\circ} \text{ kV}, \qquad \mathbf{V}_{ca} = 26\underline{/-120} \text{ kV}$$

Finally, converting to the peak voltage $\sqrt{2}(26 \text{ kV}) = 36.8$ kV, we obtain

$$v_{ab}(t) = 36.8\cos(\omega t + 120^\circ) \text{ kV}$$
$$v_{bc}(t) = 36.8\cos(\omega t) \text{ kV}$$
$$v_{ca}(t) = 36.8\cos(\omega t - 120^\circ) \text{ kV}$$

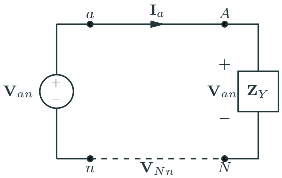
Balanced Wye Loads



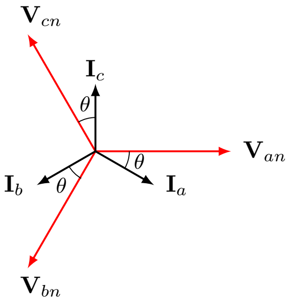
- A three-phase load consists of three impedance branches. This branches might correspond to three separate devices or to a three-phase device such as a motor with three windings.
- When the impedances are connecteed in a Wye configuration, they form a **Wye load**. The load is **balanced** if each branch has the same impedance $\mathbf{Z}_Y = |\mathbf{Z}_Y|\underline{/\theta}$.
- The balanced condition has practical importance because the total instantaneous power to the load will be constant, even when the load includes energy-storage elements as well as resistance.

Balanced Wye Loads

A three-phase circuit with a balanced Wye load can be analyzed using an equivalent circuit for just one phase. We assume that the lines from the generator are ideal conductors (no impedance). We also temporarily assume a Wye generator whose neutral point is connected to the load neutral N through an ideal conductor indicated by the dashed line.



(a)



(b)

Here $\mathbf{V}_{Nn} = 0$ and the source voltage $\mathbf{V}_{an}$ appears across the branch impedance, so $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$, $\mathbf{I}_b = \mathbf{V}_{bn}/\mathbf{Z}_Y$, and $\mathbf{I}_c = \mathbf{V}_{cn}/\mathbf{Z}_Y$.

Balanced Wye Loads

Since $|\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = V_\phi/|\mathbf{Z}_Y|$ and $V_\phi = V_l/\sqrt{3}$ all three lines carry the same **rms line current** given by

$$I_l = \frac{V_\phi}{|\mathbf{Z}_Y|} = \frac{V_l}{\sqrt{3}|\mathbf{Z}_Y|}$$

Then, taking $\underline{/\mathbf{V}_{an}} = 0°$, we have (load RL we have θ_i lag θ_v)

$$\mathbf{I}_a = I_l\underline{/-\theta}, \qquad \mathbf{I}_b = I_l\underline{/-120° - \theta}, \qquad \mathbf{I}_c = I_l\underline{/120° - \theta}$$

Note: The currents form a symmetrical three-phase set, regardless of the value of $\mathbf{Z}_Y$. Hence, the neutral wire again carries no current and it may be removed from the network. Nonetheless, the symmetry still ensures that

$$\mathbf{V}_{Nn} = 0$$

In Wye connection, the current in the line connecting the source to the load is the same as the phase current flow through the impedance $\mathbf{Z}_Y$. Therefore, in the Wye-Wye connection $I_l = I_\mathbf{Y}$, where I_l is the magnitude of the line current and $I_\mathbf{Y}$ is the magnitude of the current in the Wye-connected load.

Balanced Wye Loads

Each phase has rms voltage V_ϕ , rms current I_l and impedance $\mathbf{Z}_Y$, so the real and reactive power **per phase** are

$$P_\phi = \mathrm{Re}\{\mathbf{Z}_Y\} I_l^2 = V_\phi I_l \cos\theta$$
$$Q_\phi = \mathrm{Im}\{\mathbf{Z}_Y\} I_l^2 = V_\phi I_l \sin\theta$$

Since the generator supplies three identical phases, the **total** real and reactive power from the generator are

$$P_T = 3P_\phi = 3V_\phi I_l \cos\theta = \sqrt{3} V_l I_l \cos\theta$$
$$Q_T = 3Q_\phi = 3V_\phi I_l \sin\theta = \sqrt{3} V_l I_l \sin\theta$$

Where $V_l = \sqrt{3} V_\phi$, The total apparent power is

$$|\mathbf{S}| = \sqrt{P^2 + Q^2} = \sqrt{3} V_l I_l$$

from which pf $= P/|\mathbf{S}| = \cos\theta$. Thus, the three-phase power factor exactly equals the power factor of a single branch impedance.

Three-Phase Circuit (Wye-Wye)

An abc-sequence three-phase voltage source connected in a balanced Wye havs a line voltage of $\mathbf{V}_{ab} = 380\underline{/-30^\circ}$ Vrms. We wish to determine the phase voltages.

Solution: The magnitude of the phase voltage is given by the expression

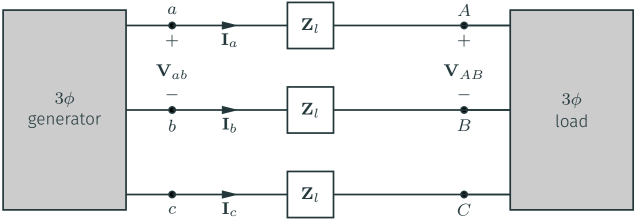
$$V_\phi = \frac{380}{\sqrt{3}} \approx 220 \text{ Vrms}$$

The phase relationships between the line and phase voltages are shown in the phasor diagram. We note that

$$\begin{aligned}
\mathbf{V}_{an} &= 220\underline{/-30^\circ - 30^\circ} = 220\underline{/-60^\circ} \\
\mathbf{V}_{bn} &= 220\underline{/-60^\circ - 120^\circ} = 220\underline{/-180^\circ} \\
\mathbf{V}_{cn} &= 220\underline{/-180^\circ - 120^\circ} = 220\underline{/60^\circ}
\end{aligned}$$

The magnitudes of these voltages are quite common and one often hears that the electric service in a building, for example, is three-phase $380/220$ V

Three-Phase Circuit with Line Impedances (Wye-Wye)



The network above depicts a three-phase transmission line connecting a high-voltage generator to a load. Each phase of the line has impedance $\mathbf{Z}_l$, and the load is a balanced Wye with branch impedance $\mathbf{Z}$. We are given that

$$|\mathbf{V}_{ab}| = 45 \text{ kV}, \qquad \mathbf{Z}_l = 0.5 + j3\ \Omega, \qquad \mathbf{Z} = 4.5 + j9\ \Omega$$

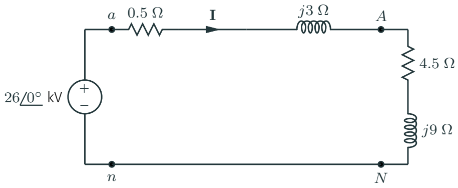
Our task is to find the rms line current and the various powers. Since the load plus transmission line presents a balanced Wye condition to the generator, we can expedite the analysis using an equivalent circuit for one phase.

Three-Phase Circuit with Line Impedances (Wye-Wye)

First, we calculate the phase voltage at the generator with $V_l = |\mathbf{V}_{ab}|$, so

$$V_\phi = \frac{45}{\sqrt{3}} = 26 \text{ kV}$$

Second, we take $\mathbf{V}_{an}$ as the reference and draw the equivalent phase-a loop in the Figure below:



The total phase impedance is

$$\mathbf{Z}_Y = \mathbf{Z}_l + \mathbf{Z} = 5 + j12 \ \Omega = 13\underline{/67.4^\circ} \ \Omega$$

Three-Phase Circuit with Line Impedances

Then, we have

$$I_l = |\mathbf{I}_a| = \frac{|\mathbf{V}_{an}|}{13} = \frac{26 \text{ kV}}{13} = 2 \text{ kA}$$

Thus

$$P = 3I_l^2(R) = 3(2 \times 10^3)(5) = 60 \text{ MW}$$
$$Q = 3I_l^2(X) = 3(2 \times 10^3)(12) = 144 \text{ MVar}$$

Note: 60 MW supplied by the generator, the power that reaches the load is $P_L = 3(4.5)(2 \times 10^3)^2 = 54$ MW, so

$$P_L/P = 54/60 = 90\%$$

The power factor need to be improve!.

Balanced Three-Phase Circuits (Delta-Connection)

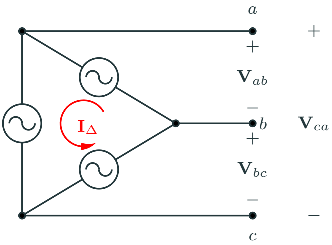
Three-phase windings can be connected in a **Delta (Δ) generator**.

- The line voltages equal the source voltages in this case, but we will continue to user $\mathbf{V}_l$ for the rms line voltage.
- The neutral point and the phase voltages do not physically exist in a Delta generator.
- The equivalent neutral point and phase voltages are defined by the accompanying phasor diagram, where $\mathbf{V}_{ab}$ has been taken as the reference. Thus,

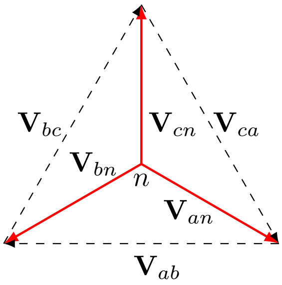
$$\mathbf{V}_{ab} = V_l \underline{/0^\circ}, \quad \mathbf{V}_{an} = (V_l/\sqrt{3})\underline{/-30^\circ} \quad \text{and so forth.}$$

- A Delta generator acts externally just like a three-terminal Wye generator with the same rms line voltage. Delta generators differ internally from Wye generators by the absence of the neutral point and the presence of the Delta mesh.
- The interior mesh current $\mathbf{I}_\Delta$ equals zero because $\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca} = 0$. However, any deviation from that voltage condition would produce a large and unwanted circulating current. Consequently, Delta generators are usually found only in special applications.

Balanced Three-Phase Circuits (Delta-Connection)



(a) Diagram of a Delta generator



(b) Phasers with $\mathbf{V}_{ab}\underline{/0^\circ}$

If the delta source are

$$\mathbf{V}_{ab} = V_l\underline{/0^\circ}, \quad \mathbf{V}_{bc} = V_l\underline{/-120^\circ}, \quad \mathbf{V}_{ca} = V_l\underline{/120^\circ},$$

where V_l is the magnitude of the line voltage.

Balanced Three-Phase Circuits (Delta-Connection)

The equivalent Wye sources are

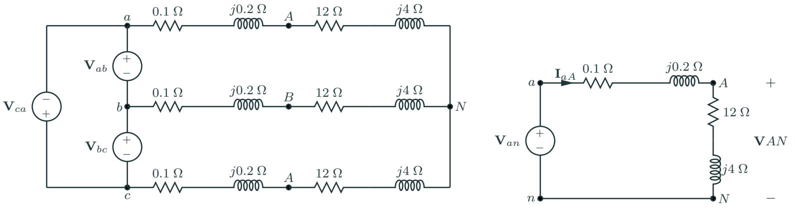
$$\mathbf{V}_{an} = \frac{V_l}{\sqrt{3}}\angle{-30^\circ} = V_\phi\angle{-30^\circ},$$
$$\mathbf{V}_{bn} = \frac{V_l}{\sqrt{3}}\angle{-150^\circ} = V_\phi\angle{-150^\circ},$$
$$\mathbf{V}_{cn} = \frac{V_l}{\sqrt{3}}\angle{-220^\circ} = V_\phi\angle{90^\circ},$$

where V_ϕ is the magnitude of the phae voltage of an equivalent Wye-connected source.

- Therefore, if we encounter a network containing a Delta-connected source, we can easily convert the source from Delta to Wye so that all the techniques we have discussed previously can be applied in the analysis.

Balanced Three-Phase Circuits (Delta-Connection)

Consider the network shown in Figure below. We wish to determine the line currents and the magnitude of the line voltage at the load. We have $\mathbf{V}_{ab} = 380\underline{/0^\circ}$, $\mathbf{V}_{bc} = 380\underline{/-120^\circ}$, and $\mathbf{V}_{ca} = 380\underline{/-240^\circ}$ Vrms.



(a) (b)

Delta-to-Wye: The single-phase diagram in (b) have $\mathbf{V}_{an} = \frac{380}{\sqrt{3}}\underline{/-30^\circ}$ Vrms. The line current $\mathbf{I}_{aA}$ is

$$\mathbf{I}_{aA} = \frac{(380/\sqrt{3})\underline{/-30^\circ}}{12.1 + j4.2} = \frac{220\underline{/-30^\circ}}{12.81\underline{/19.14^\circ}} = 17.17\underline{/-49.14^\circ}\text{ Arms}$$

and $\mathbf{I}_{bB} = 17.17\underline{/-169.146^\circ}$ Arms and $\mathbf{I}_{cC} = 17.17\underline{/70.86^\circ}$ Arms.

The voltage $\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathbf{Z}$ is then

$$\begin{aligned}\mathbf{V}_{AN} &= (17.17\underline{/-49.14^\circ})(12 + j4) = (17.17\underline{/-49.14^\circ})(12.65\underline{/18.43}) \\ &= 217.2\underline{/-30.71^\circ} \text{ Vrms}\end{aligned}$$

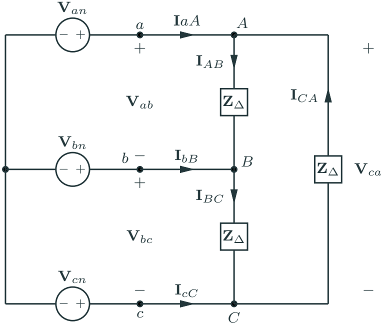
Therefore, the magnitude of the line voltage at the load ($V_l = \sqrt{3}V_\phi$) is

$$V_l = \sqrt{3}(217.2) = 376.2 \text{ Vrms}$$

The phase voltage at the source is $V_\phi = 380/\sqrt{3} = 220$ Vrms, while the phase voltage at the load is $V_\phi = 376.2/\sqrt{3} = 217.2$ Vrms.

Delta-Connected Load

Consider now the Δ-connected load show below . Note that in this connection the line-to-line voltage is the voltage across each load impedance.



The phase voltage of the source are

$$\mathbf{V}_{an} = V_\phi \angle 0^\circ$$
$$\mathbf{V}_{bn} = V_\phi \angle -120^\circ$$
$$\mathbf{V}_{cn} = V_\phi \angle 120^\circ$$

Then the line voltages are

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi \angle 30^\circ = V_l \angle 30^\circ = \mathbf{V}_{AB}, \quad \mathbf{V}_{bc} = \sqrt{3}V_\phi \angle -90^\circ = V_l \angle -90^\circ = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \sqrt{3}V_\phi \angle -210^\circ = V_l \angle -210^\circ = \mathbf{V}_{CA}$$

where V_l is the magnitude of the line voltage at both the Delta-connected load and at the source.

We note that if $\mathbf{Z}_\Delta = Z_\Delta \underline{/\theta}$, the phase currents at the load are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \mathbf{I}_\Delta$$

where $\mathbf{I}_{BC}$ and $\mathbf{I}_{CA}$ have the same magnitude but lag $\mathbf{I}_{AB}$ by $120°$ and $240°$, respectively. Using KCL, we have

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} + \mathbf{I}_{AC} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

It is more simpler to convert the balanced Δ-connected load to a balanced Y-connected load using the $\Delta - Y$ transformation

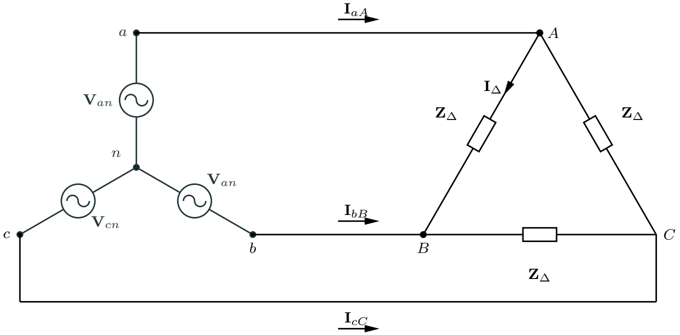
$$\mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta \quad \Longrightarrow \quad \mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \mathbf{I}_{\mathbf{Y}} = \frac{\frac{1}{\sqrt{3}}\mathbf{V}_{AB}}{\frac{1}{3}\mathbf{Z}_\Delta} = \sqrt{3}\mathbf{I}_\Delta$$

Finally, the magnitudes of the phase currents in the Δ-connected load and the line currents is

$$I_l = \sqrt{3} I_\Delta$$

Delta-Connected Load

A balanced Delta-connected load contains a 10 Ω resistor in series with a 20 mH inductor in each phase. The voltage source is an abc-sequence three-phase 50 Hz, balanced Wye with a voltage $\mathbf{V}_{an} = 220\underline{/30^\circ}$ Vrms. We wish to determine all Δ currents and line currents.



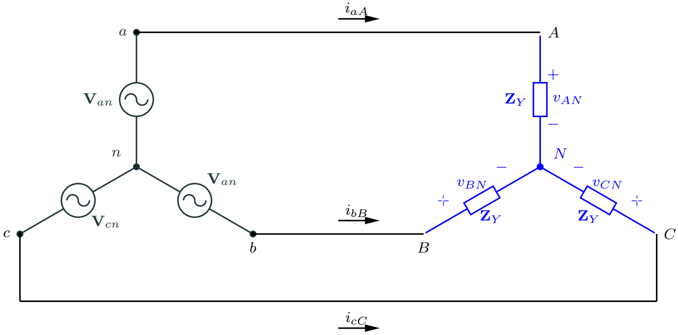
We have $j\omega L = j(50(2\pi))(20 \times 10^3) = j6.28\ \Omega$. Then the impedance per phase in the delta load is $\mathbf{Z}_{\Delta} = 10 + j6.28\ \Omega$. The line voltage $\mathbf{V}_{ab} = \sqrt{3}V_{\phi} = 220\sqrt{3}\underline{/60^\circ}$ Vrms. Since thre is no line impedance, $\mathbf{V}_{AB} = \mathbf{V}_{ab} = 220\sqrt{3}\underline{/60^\circ}$ Vrms.

Delta-Connected Load

Hence

$$\mathbf{I}_{AB} = \frac{220\sqrt{3}\underline{/60^\circ}}{10 + j6.28} = \frac{380\underline{/60^\circ}}{11.8\underline{/32.13^\circ}} = 32.18\underline{/27.87^\circ}\ \text{Arms}$$

Change Delta to Wye:



Delta-Connected Load

Since $\mathbf{Z}_\Delta = 10 + j6.28\ \Omega$, then we change them to Y-connection (blue color)

$$\mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta = 3.33 + j2.1 = 3.94\underline{/32.24^\circ}\ \Omega$$

then the line current (See Figure in the previous slide.)

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{220\underline{/30^\circ}}{3.94\underline{/32.24^\circ}} = 55.84\underline{/-2.24^\circ}\ \text{Arms}$$

Therefore, the remaining phase and line currents are

$$\begin{aligned} \mathbf{I}_{BC} &= 32.18\underline{/-92.23^\circ}\text{Arms} & \mathbf{I}_{bB} &= 55.84\underline{/-122.24^\circ}\text{Arms} \\ \mathbf{I}_{CA} &= 32.18\underline{/152.24^\circ}\text{Arms} & \mathbf{I}_{cC} &= 55.84\underline{/117.76^\circ}\text{Arms} \end{aligned}$$

Note: $I_l = \sqrt{3} I_\phi$.

Power Relationships

Weather the load is connected in a Wye or a Delta, the real and reactive power per phase is

$$P_\phi = V_\phi I_\phi \cos(\theta), \qquad Q_\phi = V_\phi I_\phi \sin(\theta),$$

where θ is the angle between the phase voltage and the line current, or

$$P_\phi = \frac{V_l I_l}{\sqrt{3}} \cos(\theta), \qquad Q_\phi = \frac{V_l I_l}{\sqrt{3}} \sin(\theta)$$

The total real and reactive power for all three phases is then

$$P_T = 3P_\phi = \sqrt{3} V_l I_l \cos(\theta), \quad Q_T = 3Q_\phi \sqrt{3} V_l I_l \sin(\theta)$$

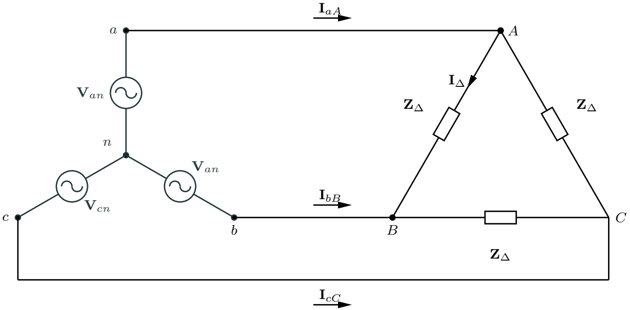
and, therefore, the magnitude of the complex power (apparent power) is

$$|\mathbf{S}_T| = \sqrt{P_T^2 + Q_T^2} = \sqrt{3} V_l I_l \quad \text{and} \quad \angle \mathbf{S}_T = \theta$$

Power Relationships: Example I

A three-phase balanced Wye-Delta system has a line voltage of 380 Vrms. The total real power absorbed by the load is 1200 W. If the power factor angle of the load is $20°$ lagging, we wish to determine the magnitude of the line current and the value of the load impedance per phase in the Delta.

Solution:



The line current can be obtained from

$$P_\phi = \frac{V_l I_l}{\sqrt{3}} \cos(\theta) = \frac{380 I_l}{\sqrt{3}} \cos(20°) = \frac{P_T}{3} = \frac{1200}{3} = 400 \text{ W}$$

$$I_l = \frac{400\sqrt{3}}{380\cos(20^\circ)} = 1.94 \text{ Arms}$$

The magnitude of the current in each leg of the Delta-connected load is

$$I_\Delta = \frac{I_l}{\sqrt{3}} = \frac{1.94}{\sqrt{3}} = 1.12 \text{ Arms}$$

Therefore, the magnitude of the Delta impedance in each phase of the load is

$$|\mathbf{Z}_\Delta| = \frac{V_l}{I_\Delta} = \frac{380}{1.12} = 339.29\ \Omega$$

Since the power factor angle is 20° lagging, the load impedance is

$$\begin{aligned}\mathbf{Z}_\Delta &= 339.29\underline{/20^\circ} \\ &= 319.82 + j116\ \Omega\end{aligned}$$

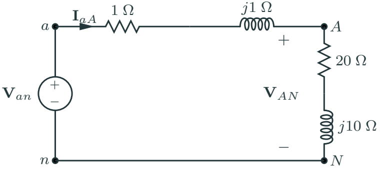
Power Relationships: Example II

A three-phase Wye-connected load is supplied by an abc-sequence balanced three-phase Wye-connected source with a phase voltage of 220 Vrms. If the line impedance and load impedance per phase are $1 + j1$ and $20 + j10\ \Omega$ respectively, we wish to determine the value of the line currents and the load voltage. Moreover we wish to determine the real and reactive power per phase at the load and the total real power, reactive power, and the complex power at the source.

Solution: The phase voltages are

$$\mathbf{V}_{an} = 220\underline{/0^\circ} \text{ Vrms}, \quad \mathbf{V}_{bn} = 220\underline{/-120^\circ} \text{ Vrms}, \quad \mathbf{V}_{cn} = 220\underline{/+120^\circ} \text{ Vrms}.$$

The per phase circuit diagram is shown below



Power Relationships: Example II

The line current for the a phase is

$$I_l = \mathbf{I}_{aA} = \frac{220\angle 0^\circ}{21 + j11} = \frac{220\angle 0^\circ}{23.71\angle 27.65} = 9.28\angle -27.65^\circ = I_\phi \angle -27.65^\circ$$

The load voltage for the a phase, which we call $\mathbf{V}_{aN}$, is

$$\mathbf{V}_{AN} = 9.28\angle -27.65^\circ (20 + j10) = 9.28\angle -27.65^\circ (22.36\angle 26.57^\circ) = 207.50\angle -1.08^\circ \text{ Vrms}$$

The corresponding line currents and load voltages for the b and c phases are

$$\begin{aligned} \mathbf{I}_{bB} &= 9.28\angle -147.65^\circ \text{ Arms} & \mathbf{V}_{BN} &= 207.50\angle -121.08^\circ \\ \mathbf{I}_{cC} &= 9.28\angle -267.65^\circ \text{ Arms} & \mathbf{V}_{CN} &= 207.50\angle -241.08^\circ \end{aligned}$$

From the data above the complex power per phase at the load is

$$\begin{aligned} \mathbf{S}_{\text{load}} &= \mathbf{V}\mathbf{I}^* = (207.50\angle -1.08^\circ)(9.28\angle 27.65^\circ) = 1925.6\angle 26.57^\circ \\ &= 1722.23 + j861.30 \text{ VA} = P_\phi + jQ_\phi \text{ VA} \end{aligned}$$

Power Relationships: Example II

Therefore, the real and reactive power per phase at the load are 1722.23 W and 861.30 Var, respectively.
The complex poser per phase at the source is

$$\mathbf{S}_{\text{source}} = \mathbf{VI}^* = (220\underline{/0^\circ})(9.28\underline{/27.65^\circ}) = 2041.6\underline{/27.65}$$
$$= 1808.45 + j947.44 \text{ VA} = P_{\text{source}} + jQ_{\text{source}} \text{ VA}$$

Therefore, total real power, reactive power, and apparent power at the source are

$$P_T = (3)(1808.45) = 5425.35 \text{ W}, \quad Q_T = (3)(947.44) = 2842.32 \text{ Var}$$
$$S_T = (3)2041.6 = 6124.8 \text{ VA}$$

Reference

1. William H. Hayt, Jr., Jack E. Kemmerly, and Steven M. Durbin *Engineering Circuit Analysis*, 8th Edition McGraw-Hill, 2012.
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3. A. Bruce Calson *Circuits: Enginering Concepts and Analysis of Linear Electric Circuits*, 1st Edition, Thomson-Engineering, 1999