RC2017 Signals and Systems **Homework 9**
Assign: Thu 5 April 2019 **Due for RC2: Wed 10 April 2019**

Instruction: Hand in your work with name and code to my desk by 10.00 am. of the due date. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. Find the Fourier Transform of the signal $f(t)$ in Fig. 1 (a) and (b). (10 points)

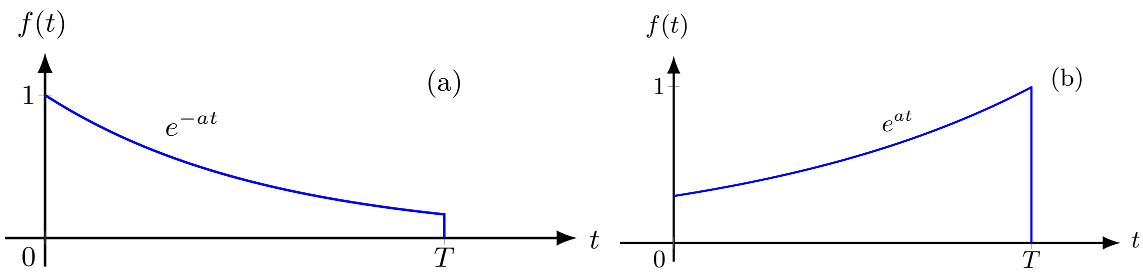

Figure 1: Problem 2

Solution:
(a) We have

$$F(\omega) = \int_0^T e^{-at}e^{-j\omega t}dt = -\frac{1}{j\omega + a}\, e^{-(a+j\omega)t}\Big|_0^T$$
$$= \frac{1}{j\omega + a}\left(1 - e^{-(j\omega+a)T}\right)$$

□

(b) We have

$$F(\omega) = \int_0^T e^{at}e^{-j\omega t}dt = \frac{1}{j\omega - a}\left(1 - e^{-(j\omega-a)T}\right)$$

□

2. Find the inverse Fourier transforms of the spectra in Fig. 3 (a) and (b) (10 points)
Solution:

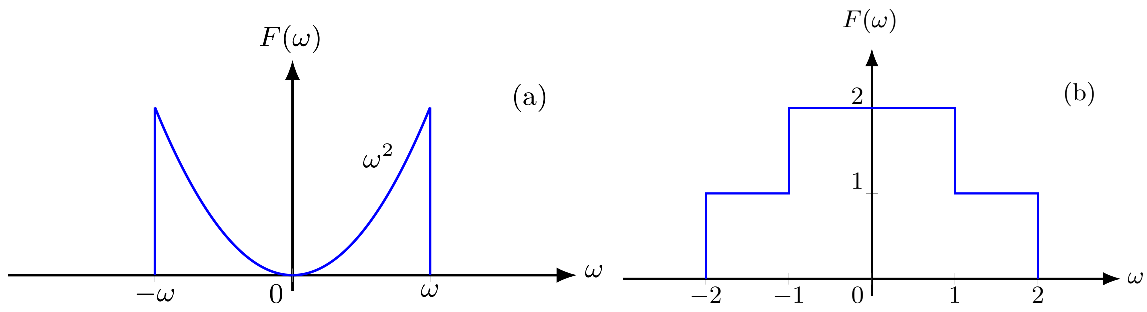

Figure 2: Problem 3

(a) We have

$$f(t) = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} \omega^2 e^{j\omega t} d\omega$$

Using by-part integral two times, we have

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \left[\frac{\omega_0}{jt} \left(e^{j\omega_0 t} - e^{-j\omega_0 t} \right) + \frac{2\omega_0}{t^2} \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right) - \frac{2}{jt^3} \left(e^{j\omega_0 t} - e^{-j\omega_0 t} \right) \right] \\ &= \frac{1}{\pi t^3} \left[\omega_0 t^2 \sin \omega_0 t + 2\omega_0 t \cos \omega_0 t - 2 \sin \omega_0 t \right] \\ &= \frac{1}{\pi t^3} \left[\left(\omega_0^2 t^2 - 2 \right) \sin \omega_0 t + 2\omega_0 t \cos \omega_0 t \right] \end{aligned}$$

$\square$

(b) We can separate the $F(\omega)$ into $F_1(\omega) + F_2(\omega)$ as shown below:

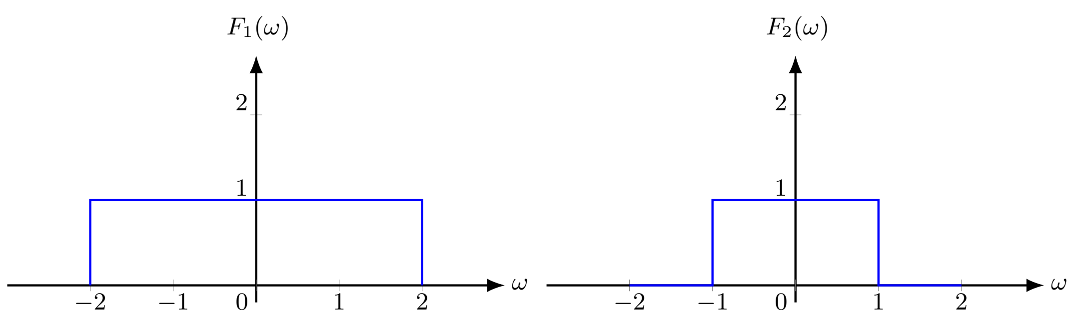

Figure 3: Problem 3

$$
\begin{aligned}
f_1(t) &= \frac{1}{2\pi}\int_{-2}^{2} e^{j\omega t} d\omega = \frac{1}{2\pi}\left[\frac{1}{jt}\left(e^{j2t} - e^{-j2t}\right)\right] = \frac{\sin 2t}{\pi t} \\
f_2(t) &= \frac{1}{2\pi}\int_{-1}^{1} e^{j\omega t} d\omega = \frac{1}{2\pi}\left[\frac{1}{jt}\left(e^{jt} - e^{-jt}\right)\right] = \frac{\sin t}{\pi t} \\
f(t) &= f_1(t) + f_2(t) = \frac{\sin 2t + \sin t}{\pi t}
\end{aligned}
$$

$\square$