RC2017 Signals and Systems **Homework 8**
Assign: Fri 29 March 2019 **Due for RC2: Wed 3 April 2019**

Instruction: Hand in your work with name and code to my desk by 10.00 am. of the due date. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. Determine the Exponential Fourier Series coefficients for the signal $\varphi(t)$ depicted in Fig. 1 and also plot the Magnitude and phase spectra of the signal.
   Solution: Firstly, find the FS coefficients of e^{-2t} instead of $e^{-2(t-1)}$ with $T_0 = 2$.

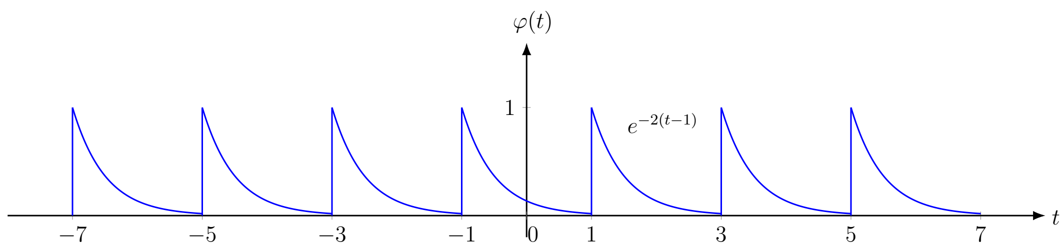


Figure 1: Time-domain signal for the question

We have

$$D_n = \frac{1}{2}\int_0^2 e^{-2t}e^{-jn\omega_0 t}dt = \frac{1}{2}\frac{-1}{jn\omega_0+2}e^{-(jn\omega_0+2)t}\Big|_0^2$$
$$= \frac{-0.5}{jn\omega_0+2}\left[e^{-(j2n\omega_0+4)}-1\right]$$

Since, $\omega_0 = 2\pi/T_0 = \pi$, we have

$$D_n = \frac{-0.5}{jn\pi+2}\left[e^{-4}-1\right] = \frac{0.49}{jn\pi+2}, \qquad \text{note}: \ e^{-j2n\pi} = 1$$

The signal $e^{-2(t-1)}u(t-1)$ is the $e^{-2t}u(t)$ with a delay by $T/2 = 1$ second, so we have the FS coefficients of this function as

$$\hat{D}_n = D_n e^{-jn\omega_0 T/2} = D_n e^{-jn\omega_0} = \frac{0.49e^{-jn\pi}}{jn\pi+2}$$

Then

$$\hat{D}_0 = 0.245, \qquad \hat{D}_1 = \frac{0.49}{j\pi+2}e^{-j\pi} = \frac{-0.49}{j\pi+2}, \qquad \hat{D}_{-1} = \frac{-0.49}{-j\pi+2}$$

$$\hat{D}_2 = \frac{0.49}{j2\pi+2}e^{-j2\pi} = \frac{0.49}{j2\pi+2} \qquad \hat{D}_{-2} = \frac{0.49}{-j2\pi+2},$$

$$\hat{D}_3 = \frac{0.49}{j3\pi+2}e^{-j3\pi} = \frac{-0.49}{j3\pi+2} \qquad \hat{D}_{-3} = \frac{-0.49}{-j3\pi+2},$$

$$\hat{D}_4 = \frac{0.49}{j4\pi+2}e^{-j4\pi} = \frac{0.49}{j4\pi+2} \qquad \hat{D}_{-4} = \frac{0.49}{-j4\pi+2},$$

$$\hat{D}_5 = \frac{0.49}{j5\pi+2}e^{-j5\pi} = \frac{-0.49}{j5\pi+2} \qquad \hat{D}_{-5} = \frac{0.49}{-j5\pi+2}$$

The magnitude and phase spectrum are shown in the table 1.

Table 1: Magnitude and Phase Spectrum

No.	0	$-1, 1$	$-2, 2$	$-3, 3$	$-4, 4$	$-5, 5$
$\lvert D_n \rvert$	0.245	0.1316	0.0743	0.0509	0.0385	0.0309
$\angle D_n$	$0°$	$\mp 122.48°$	$\pm 72.34°$	$\mp 78°$	$\pm 80.95°$	$\mp 82.74°$

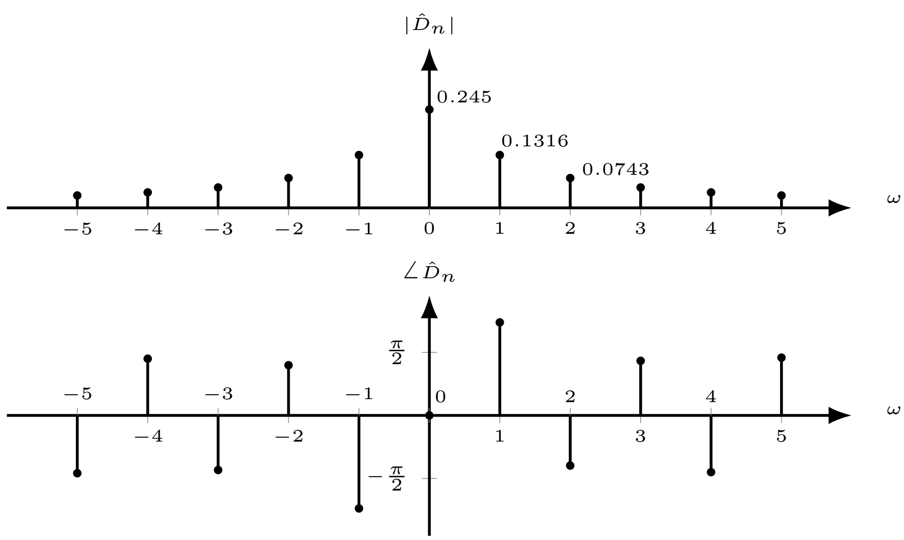
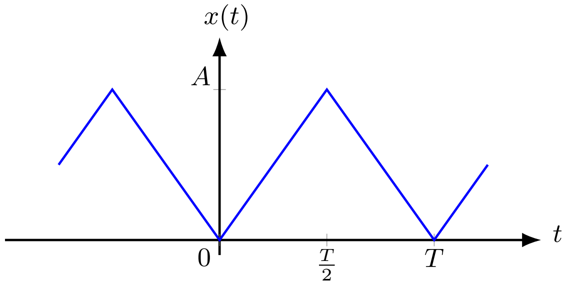


Figure 2: Magnitude and Phase Spectrum plots

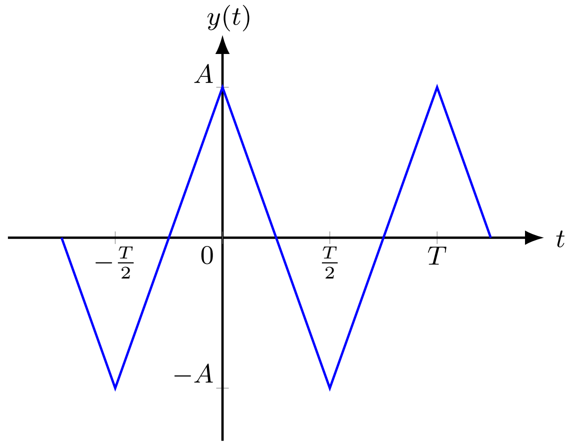
□

2. If the Fourier coefficients D_n of the signal $x(t)$ in Figure 3a is given by

$$D_n = \begin{cases} \frac{A}{2}, & n = 0 \\ \frac{-2A}{\pi^2 n^2}, & n \text{ odd} \\ 0, & \text{otherwise} \end{cases}$$



(a)



(b)

Figure 3: Problem 3

Find the Fourier coefficients D_n of the signal $y(t)$ shown in Figure 3b. (10 points)
Solution: From 3 and **??**, it appears that $y(t) = 2x(t + \frac{T}{2}) - A$. The time shift is $t_0 = -\frac{T}{2}$ so

$$e^{-jn\omega t_0} = e^{-jn\frac{2\pi}{T}(-\frac{T}{2})} = e^{j\pi n} = (-1)^n.$$

The the corresponding coefficients under the transformation, $D_n \to Y_n$ is

$$Y_n = \begin{cases} 2D_0 - A, & n = 0 \\ 2D_n(-1)^n & n \neq 0 \end{cases}.$$

Note $(-1)^0 = 1$. Then for $n = 0$

$$2D_0 - A = 2\frac{A}{2} - A = 0$$

and for n is odd:

$$2D_n(-1)^n = 2\frac{-2A}{\pi^2 n^2}(-1) = \frac{4A}{\pi^2 n^2}$$

This reveals your solution is

$$Y_n = \begin{cases} 0, & n \text{ even} \\ \frac{4A}{\pi^2 n^2}, & n \text{ odd} \end{cases}$$

$\square$