RC2017 Signals and Systems **Homework 7**

Assign: Thu 21 March 2019 **Due for RC2: Wed 27 March 2019**

Instruction: Hand in your work with name and code to my desk by 10.00 am. of the due date. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. In Figure 1 $R_1 = 3\ \Omega$, $R_2 = 2\ \Omega$, $L = 1$ H, and $C = 1$ F. Find $v(t)$ and $i(t)$ given

$$i_g = \begin{cases} 1 \quad \text{A}, & t < 0 \\ \cos t, & t > 0 \end{cases}$$

Solution: For $t < 0$, we have the DC circuit shown in Figure 2

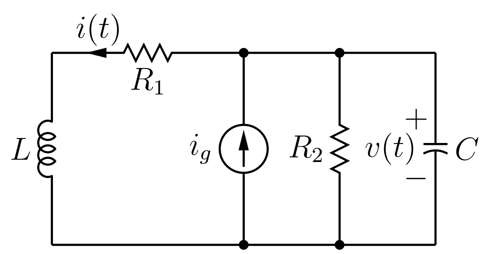

Figure 1: RLC circuit used in the Problem

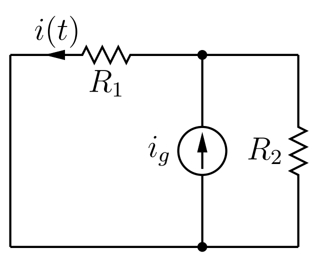

Figure 2: the equivalent DC circuit for $t < 0$

We have $i_L(0^-) = 0.4$ A and $v_{R_2}(0^-) = v(0^-) = v_C(0^-) = 1.2$ V. The Laplace transform of the circuit is shown in Figure 3.

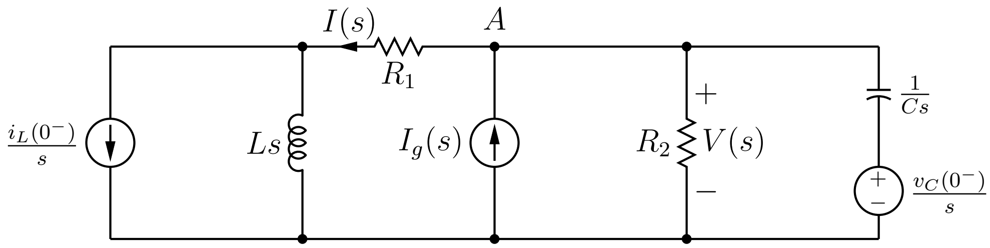

Figure 3: Laplace transform model $t > 0$

Using KCL at node A, we have

$$\begin{aligned}
I(s) + \frac{V(s)}{R_2} + \left(V(s) - \frac{v_C(0^-)}{s}\right)s &= I_g(s) \\
I(s) + \frac{V(s)}{2} + \left(V(s) - \frac{1.2}{s}\right)s &= I_g(s) = \frac{s}{s^2+1} \\
V(s)\left(s + \frac{1}{2}\right) + I(s) &= \frac{1.2s^2 + s + 1.2}{s^2+1}
\end{aligned}$$

Using KVL around the external loop, we have

$$\begin{aligned}
R_1 I(s) + \left(I(s) - \frac{i_L(0^-)}{s}\right)Ls &= V(s) \\
3I(s) + \left(I(s) - \frac{0.4}{s}\right)s &= V(s) \\
-V(s) + I(s)\,(s+3) &= 0.4
\end{aligned}$$

Rearranging to matrix from, we have

$$\begin{bmatrix} s + \frac{1}{2} & 1 \\ -1 & (s+3) \end{bmatrix} \begin{bmatrix} V(s) \\ I(s) \end{bmatrix} = \begin{bmatrix} \frac{1.2s^2+s+1.2}{s^2+1} \\ 0.4 \end{bmatrix}$$

We have

$$\begin{bmatrix} V(s) \\ I(s) \end{bmatrix} = \frac{1}{s^2 + 3.5s + 2.5} \begin{bmatrix} (s+3) & -1 \\ 1 & (s+0.5) \end{bmatrix} \begin{bmatrix} \frac{1.2s^2+s+1.2}{s^2+1} \\ 0.4 \end{bmatrix}$$

, then

$$\begin{aligned}
V(s) &= \frac{(s+3)(1.2s^2+s+1.2) - 0.4(s^2+1)}{(s^2+3.5s+2.5)(s^2+1)} \\
&= \frac{0.6667}{s+1} - \frac{0.0184}{s+2.5} + \frac{0.5513s + 0.621}{s^2+1} \\
I(s) &= \frac{(1.2s^2+s+1.2) + 0.4(s+0.5)(s^2+1)}{(s^2+3.5s+2.5)(s^2+1)} \\
&= \frac{0.3332}{s+1} - \frac{0.0367}{s+2.5} + \frac{0.1034s + 0.2414}{s^2+1}
\end{aligned}$$

Using the inverse Laplace transform, we have

$$\begin{aligned} v(t) &= 0.6667e^{-t} - 0.0184e^{-2.5t} + 0.8304\cos(t - 48.4^\circ),\ t > 0 \\ i(t) &= 0.3332e^{-t} - 0.0367e^{-2.5t} + 0.2626\cos(t - 66.8^\circ),\ t > 0 \end{aligned}$$

$\square$