RC2017 Signals and Systems | **Homework 6**
Assign: Fri 2 March 2019 | **Due for RC2: None**

Instruction: Hand in your work with name and code to my desk by 10.00 am. of the due date. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. Calculate the inverse Laplace transform of the following:

   (a) $X_1(s) = \dfrac{s+3}{s(s+1)(s+2)}$ (5 points)
   Solution:

$$
\begin{aligned}
F(s) &= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} \\
A &= \left.\frac{s+3}{(s+1)(s+2)}\right|_{s=0} = \frac{3}{2} \\
B &= \left.\frac{s+3}{s(s+2)}\right|_{s=-1} = -2 \\
C &= \left.\frac{s+3}{s(s+1)}\right|_{s=-2} = \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
F(s) &= \frac{3/2}{s} + \frac{-2}{(s+1)} + \frac{1/2}{(s+2)} \\
f(t) &= \frac{3}{2} - 2e^{-t} + \frac{1}{2}e^{-2t},\ t \geq 0
\end{aligned}
$$

$\square$

   (b) $X_2(s) = \dfrac{s+5}{s^3+5s^2+17s+13}$ (5 points)
   Solution With some trial-and-error we have

$$
\begin{aligned}
X_2(s) &= \frac{s+5}{(s+1)(s^2+4s+13)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4s+13} \\
A &= \left.\frac{s+5}{s^2+4s+13}\right|_{s=-1} = \frac{4}{10}
\end{aligned}
$$

Using short-cut method,

$$\lim_{s=\infty} s\frac{s+5}{(s+1)(s^2+4s+13)} = \lim_{s=\infty} s\left(\frac{4/10}{s+1} + \frac{Bs+C}{s^2+4s+13}\right)$$
$$0 = \frac{4}{10} + B \Rightarrow B = -\frac{4}{10}$$

Substituting $s = 0$ to the original $F(s)$ and its partial fraction, we have

$$\frac{5}{13} = \frac{4}{10} + \frac{C}{13}$$
$$C = -\frac{2}{10}$$

Then,

$$\begin{aligned} F(s) &= \frac{4/10}{s+1} + \frac{-(4/10)s - 2/10}{s^2+4s+13} \\ &= \frac{4/10}{s+1} - \left(\frac{4}{10}\frac{s+2}{(s+2)^2+3^2} - \frac{2}{10}\frac{3}{(s+2)^2+3^2}\right) \\ f(t) &= \frac{4}{10}e^{-t} - \frac{4}{10}e^{-2t}\cos(3t) + \frac{2}{10}e^{-2t}\sin(3t), t \geq 0 \end{aligned}$$

□

(c) $F(s) = \dfrac{e^{-(s-1)}+3}{s^2-2s+5}$ (5 points)

Solution

$$F(s) = \frac{e^{-(s-1)}}{s^2-2s+5} + \frac{3}{s^2-2s+5} = \frac{(1/2)e^{-(s-1)}}{(s-1)^2+2^2} + \frac{3/2}{(s-1)^2+2^2}$$
$$f(t) = \frac{1}{2}e^{t-1}\sin(2(t-1)) + \frac{3}{2}e^{t}\sin(2t),\ t \geq 0$$

□

2. Find the Laplace transforms of the following functions:

(a) $te^{-1}\mathbb{1}(1)$ (5 points)

Solution From

$$tf(t) \quad \overset{\mathcal{L}}{\Longleftrightarrow} \quad -\frac{dF(s)}{ds}$$

Then, we have

$$te^{-1} \quad \overset{\mathcal{L}}{\Longleftrightarrow} \quad -e^{-1}\frac{d}{ds}\frac{1}{s} = \frac{e^{-1}}{s^2}$$

□

(b) $\cos(\omega_1 t)\cos(\omega_2 t)\mathbb{1}(t)$ (5 points)
Solution Since

$$\cos(\omega_1 t)\cos(\omega_2 t) = \frac{1}{2}\left[\cos(\omega_1 t + \omega_2 t) + \cos(\omega_1 t - \omega_2 t)\right]$$

then, we have

$$F(s) = \frac{1}{2}\left[\frac{s}{s^2 + (\omega_1 + \omega_2)^2} + \frac{s}{s^2 + (\omega_1 - \omega_2)^2}\right]$$

$\square$

(c) $e^{-2t}\cos(5t+\theta)\mathbb{1}(t)$ (5 points)
Solution Since

$$\cos(5t+\theta) = \cos(5t)\cos\theta - \sin(5t)\sin\theta$$

then we have

$$F(s) = \cos\theta\frac{s+2}{(s+2)^2+5^2} - \sin\theta\frac{5}{(s+2)^2+5^2}$$

$\square$