RC2017 Signals and Systems **Homework 4**
Assign: Thu 14 Feb 2019 **Due for RC2: Wed 20 Feb 2019**

Instruction: Hand in your work with name and code to my desk by 10.00 am. of the due date. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. An LTIC system is specified by the block diagram in Fig. 1 For each input $f(t)$ and

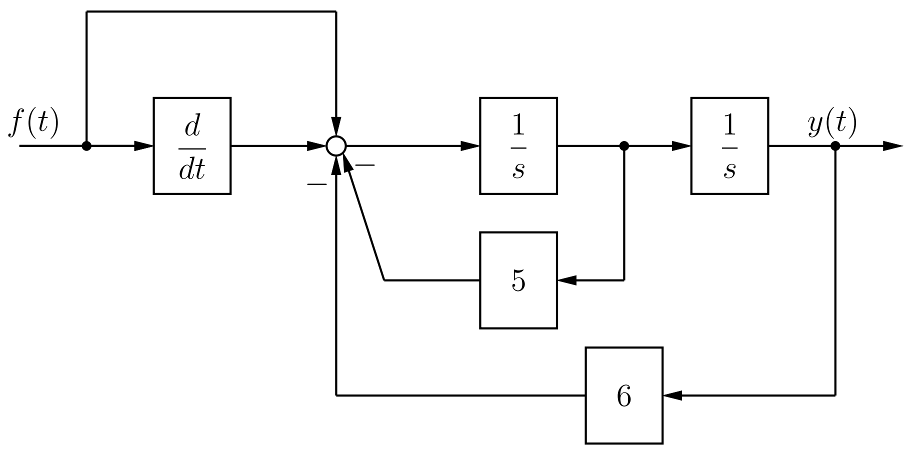

Figure 1: a block diagram of the LTIC system.

the initial values listed below, find zero-input , and impulse responses of the system.

a. $f(t) = 6t^2$, $y(0) = 2$ and $\dot{y}(0) = -1$. (5 points)
Solution: From the block diagram the system description is

$$\left(D^2 + 5D + 6\right) = (D+1)\, f(t)$$

The zero-input response is follow from:

$$\begin{aligned} \lambda^2 + 5\lambda + 6 &= 0 \\ \lambda &= -2, -3 \\ y_0(t) &= C_1 e^{-2t} + C_2 e^{-3t}, t \geq 0 \end{aligned}$$

By using given initial condition, we obtain

$$C_1 + C_2 = 2 \text{ and } -2C_1 - 3C_2 = -1$$

,then $C_1 = 5$ and $C_2 = -3$. Finally

$$y_0(t) = 5e^{-2t} - 3e^{-3t}, t \geq 0$$

The impulse response $h(t)$ is

$$h(t) = b_m \delta(t) + [P(D)y_n(t)]\mathbb{1}(t).$$

$y_n(t)$ has the same form like $y_0(t)$ but $y_n(0) = 0$ and $\dot{y}_n(0) = 1$, then

$$\begin{aligned} y_n(t) &= 2e^{-2t} - e^{-3t}, t > 0 \\ P(D)y_n(t) = (D+1)y_n(t) &= -e^{-2t} + 2e^{-3t} \end{aligned}$$

Since $b_m = 0$ in this case, hence

$$h(t) = -e^{-2t} + 2e^{-3t}, t \geq 0$$

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b. $f(t) = 4e^{-4t}$, $y(0) = 2$ and $\dot{y}(0) = -1$. (5 points)
Solution: The system and all initial conditions are the same like the previous question so the zero-input and impulse response are the same.

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2. Determine the unit impulse response of LTI systems described by

$$(D+2)y(t) = (3D+5)f(t)$$

(5 points)
Solution: The impulse response is

$$h(t) = b_n \delta(t) + [P(D)y_n(t)]\mathbb{1}(t)$$

Since $P(D) = 3D + 5$, we have $b_n = 3$. The characteristic equation is

$$\lambda + 2 = 0 \text{ or } \lambda = -2$$

, then

$$y_n(t) = Ce^{-2t}$$

From the initial condition of $y_n(t)$, we have $y_n(0^-) = 1$ or $C = 1$. Then

$$y_n(t) = e^{-2t}$$

and

$$\begin{aligned} h(t) &= 3\delta(t) + [P(D)y_n(t)]\mathbb{1}(t) \\ &= 3\delta(t) + (3D+5)e^{-2t}\mathbb{1}(t) \\ &= 3\delta(t) - 6e^{-2t} + 5e^{-2t},\ t \geq 0 \\ &= 3\delta(t) - e^{-2t} \end{aligned}$$

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