RC2018 Signals and Systems **Homework 2**
Assign: Thu 31 Jan 2019 **Due for 2RC: Wed 6 Feb 2019**

Instruction: DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. For a signal $f(t)$ shown in Fig. 1,

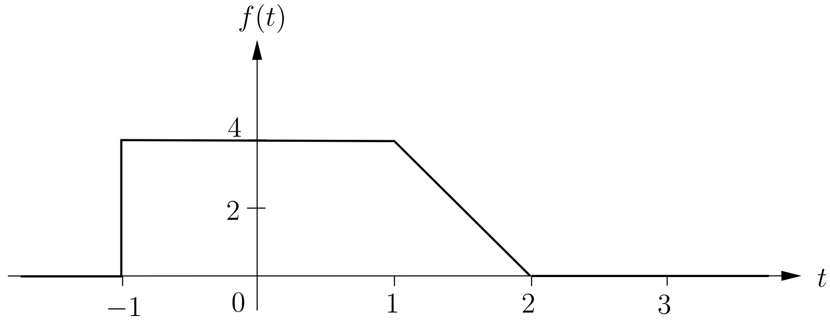
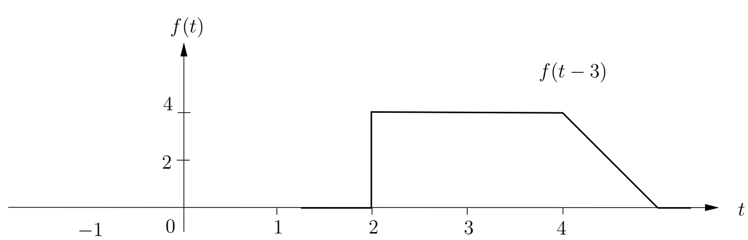
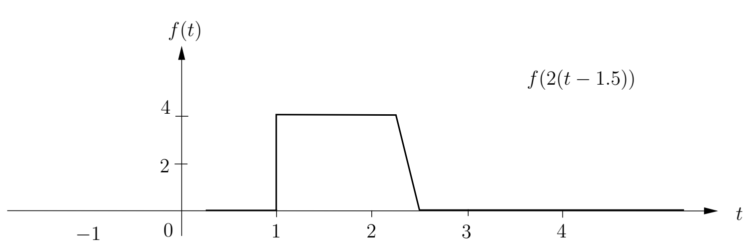


Figure 1: a signal $f(t)$ for the question 3

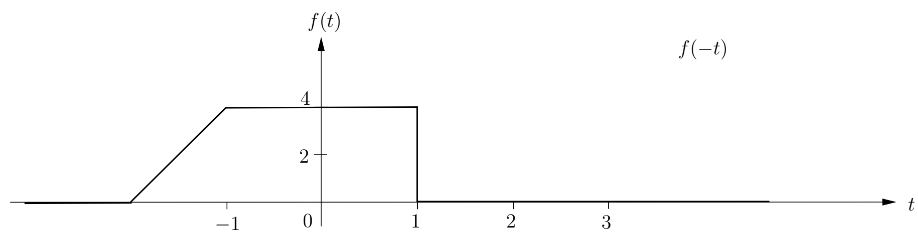
(a) Sketch signals $f_1(t) = f(2t-3)$ and $f_2(t) = f(2-t)$ (5 points)
Solution:
For $f_1(t) = f(2t-3) = f(2(t-1.5))$ we have

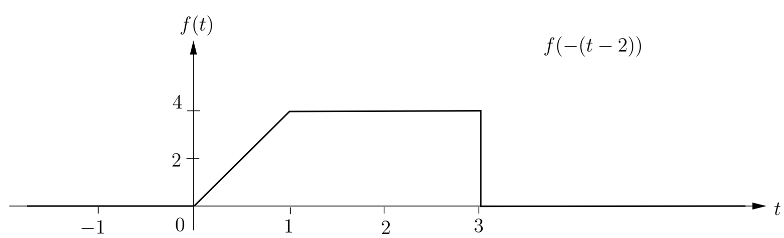




For $f_2(t) = f(2-t) = f(-(t-2))$ we have

$\square$





2. Evaluate the following integrals: (1 point for each)

(a) $\int_{-\infty}^{\infty} \delta(\tau) f(t-\tau) d\tau$

(b) $\int_{-\infty}^{\infty} f(\tau) \delta(t-\tau) d\tau$

(c) $\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$

(d) $\int_{-\infty}^{\infty} \delta(t+3) e^{-t} dt$

(e) $\int_{-\infty}^{\infty} (t^3+4) \delta(1-t) dt$

(f) $\int_{-\infty}^{\infty} e^{x-1} \cos\left[\frac{\pi}{2}(x-5)\right] \delta(x-3) dx$

Hint: $\delta(x)$ is located at $x=0$. For example, $\delta(1-t)$ is located at $1-t=0$, and so on.

Solution:

From the sampling property: $\int_{-\infty}^{\infty} f(t)\delta(t)dt = 0$

(a) The impulse is located at $\tau = 0$ and $f(t-\tau)$ at $\tau = 0$. Therefore

$$\int_{-\infty}^{\infty} \delta(\tau) f(t-\tau) d\tau = f(t)$$

(b) The impulse is located at $\tau = t$ and $f(\tau)$ at $\tau = t$. Therefore

$$\int_{-\infty}^{\infty} f(\tau) \delta(t-\tau) d\tau = f(t)$$

(c) $e^{-j\omega(0)} = 1$

(d) $e^{-(-3)} = e^3$

(e) Since $1-t=0$ when $t=1$, then $(t^3+4)\big|_{t=1} = 5$

(f) $e^{x-1}\cos[\frac{\pi}{2}(x-5)\big|_{x=3} = e^{-2}\cos(-\pi) = -e^2$

□

3. For a signal $f(t)$ shown in Fig. 1, Show a mathematic equation that describes the signal $f(t)$ in terms of unit step function $\mathbb{1}(t)$. (5 points)

Solution:

$$\begin{aligned}
f_1(t) &= 4\left[\mathbb{1}(t+1) - \mathbb{1}(t-1)\right] \\
f_2(t) &= (-4t+8)\left[\mathbb{1}(t-1) - \mathbb{1}(t-2)\right] \\
f(t) &= f_1(t) + f_2(t) \\
&= 4\mathbb{1}(t+1) + (-4t+4)\mathbb{1}(t-1) - (-4t+8)\mathbb{1}(t-2)
\end{aligned}$$

□