# INC 693, 481 Dynamics System and Modelling: Linear Graph Modeling I

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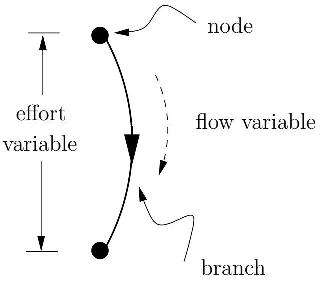
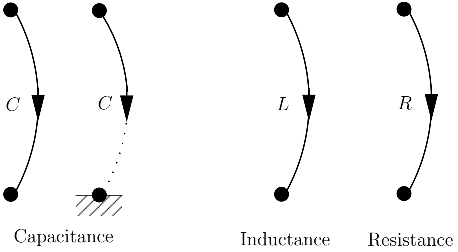
# Introduction to Linear Graph Models




graph is constructed from:

- A set of *branches* that each represent an energy port associated with a passive of source system element.
- A set of *nodes* that represent the points of interconnection of the lumped elements.

# Introduction to Linear Graph Models



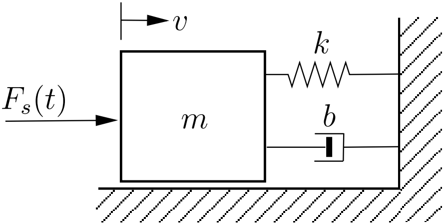

graph is constructed from:

- Capacitance elements (except electrical capacitance) must have their effort variable defined with respect to a constant reference value:
  - velocity on mass with respect to a constant velocity inertial reference frame
  - in fluid system the pressure is measured with respect to the reference pressure.

# Introduction to Linear Graph Models

- The arrow on the graph element of each type is drawn in the direction for which:
  - $e$ the effort variable associated with the branch.
  - the flow variable $f$ is defined as having a positive value.
- For source elements the arrow associated with the branch designates the sign associated with the source variable:
  - for a flow source the arrow designates the direction defined for positive flow variable flow, and
  - for an effort source the arrow designates the direction defined for the effort variable drop.
- the arrow on an effort variable source branch is commonly drawn toward the reference node.

# Introduction to Linear Graph Models



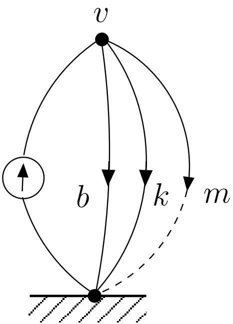



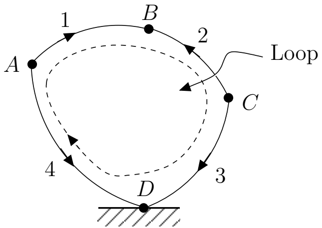

Figure: a simple mechanical system

# Element Interconnection Laws

## Compatibility

- **Compatibility law** represents a set of constraints on effort variables in the graph that may be related to physical laws. The sum of the effort variable drops on the branches around any closed loop in linear graph is identically zeros, or:

$$\sum_{i=1}^{N} e_i = 0 \text{ for any } N \text{ forming a closed loop}$$




$$\sum_{i=1}^{4} e_i = e_1 - e_2 + e_3 - e_4$$
$$= (e_A - e_B) - (e_C - e_B)$$
$$+ (e_C - e_D) - (e_A - e_D) = 0$$

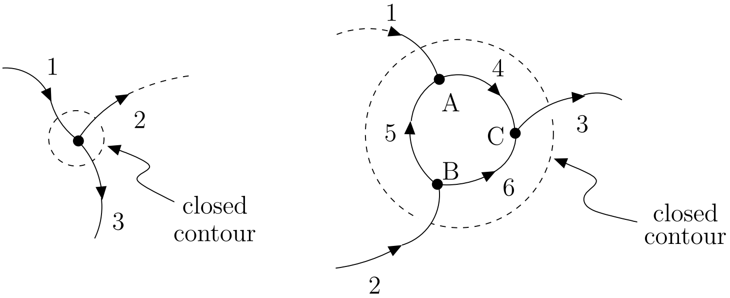
# Element Interconnection Laws
## Compatibility

The physical interpretation of the compatibility law in the various energy domains is:

- **Mechanical systems:** the velocity drops across all elements sum to zero around any closed path path in a linear graph.
- **Electrical systems:** the compatibility law is identical to KVL which states that the summation of all voltage drops around any closed loop in an electrical circuit is identically zero.
- **Fluid systems:** Pressure is a scalar potential which must sum to zero around any closed path in a fluid system.
- **Thermal systems:** Themperature is a scalar potential which must sum to zero around any closed path in a thermal system.

- **Continuity** law specifies constraints on the flow variables in a linear graph that may be related to physical laws. The sum of flow variables flowing into any closed contour drawn on a linear graph is zeros, or

$$\sum_{i=1}^{N} f_i = 0 \text{ for any } N \text{ branches that intersect a closed contour}$$




for the right hand figure:

$$f_1 - f_4 + f_5 = 0, \quad f_2 - f_5 - f_6 = 0, \quad -f_3 + f_4 + f_6 = 0$$

for the closed contour:

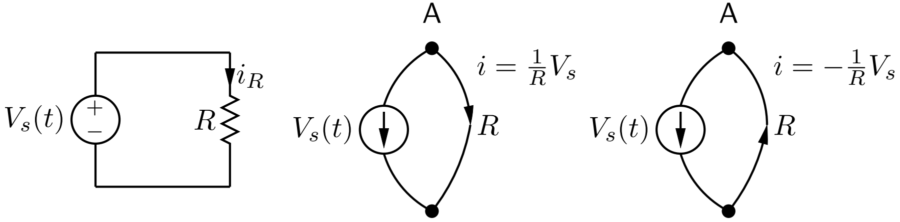
$$f_1 + f_2 - f_3 = (f_4 - f_5) + (f_5 + f_6) - (f_4 + f_6) = 0$$

The principle of continuity corresponds to the following physical constraints:

- **Mechanical systems:** In a translational (or rotatinal) mechanical system continuity at a node arises as a direct expression of Newton's laws of motion.
- **Electrical systems:** the continuity at an electrical node is KCL which states that the sum of currents flowing into any node in a circuit must be zero.
- **Fluid system:** the sum of volume flow rates into the junction must be zero.
- **Thermal systms:** the continuity of heat flow rate ensures that there is no accumulation of heat at any junction between elements.

# Sign Conventions on One-Port System Elements

## Electrical Circuit




(a) electrical system

the compatibility equation for the middle graph is

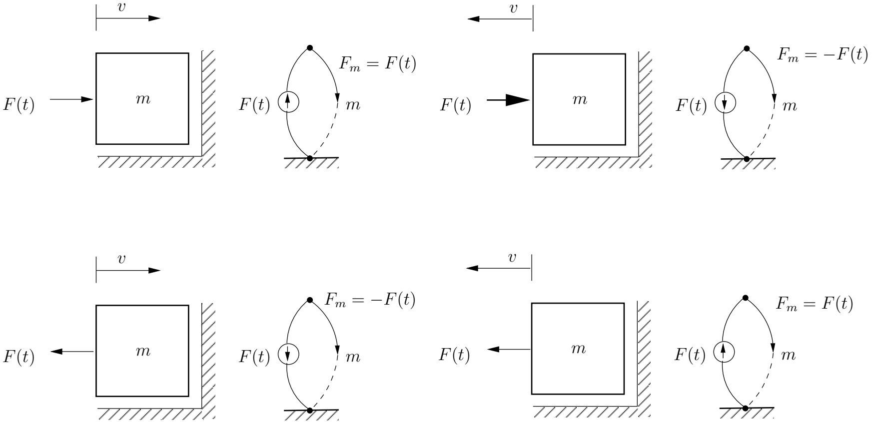
$$-V_s + v_R = 0 \quad \text{then} \quad i_R = \frac{1}{R}V_s$$

the compatibility equation for the right graph is

$$V_s + v_R = 0 \quad \text{then} \quad i_R = -\frac{1}{R}V_s.$$

# Sign Conventions on One-Port System Elements

## Mechanical System

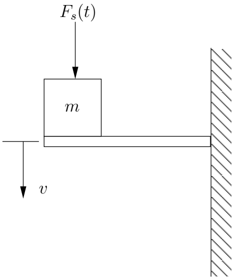


# Procedure of System Graph

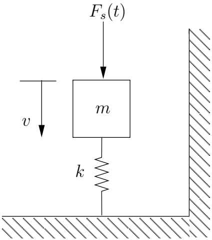
## General Procedure

1. defined the system: inputs, outputs, energy and required elements
2. for a schematic,or pictorial, model of the physical system
3. determine the system sources, energy storage and dissipation elements
4. Identify the effort variables and draw a set of nodes
5. determine the appropriate nodes for each lumped element, and inert each element into the graph
6. select a set of sign conventions for the passive elements and draw the arrows on the graph
7. select the sign conventions for the system source elements

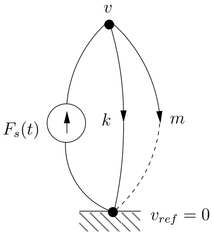
# Mechanical Translational System Models




Physical system




Schematic representation




Linear graph

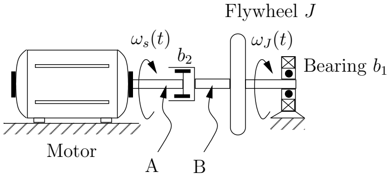
**compatibility:**

$$v_m = v_k$$

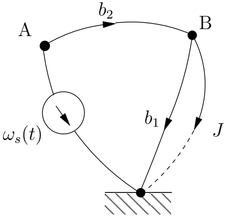
**continuity**

$$F_s - F_k - F_m = 0$$

# Mechanical Rotational System Models



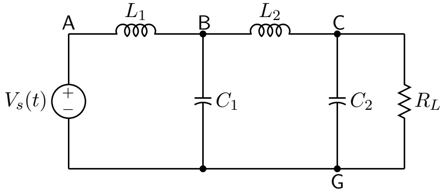

Physical system



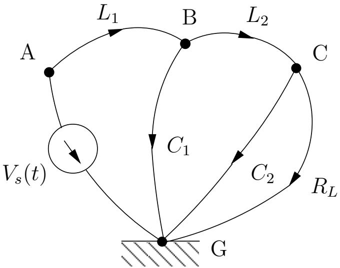

Linear graph

- We have an angular velocity source $\omega_s(t)$, a rotary inertia $J$, a rotary damper $b_1$ and a rotary damper $b_2$
- the angular velocity $\omega_J$ of flywheel must be defined relative to the fixed reference node.
- the inner bearing rotates at the same angular velocity as the flywheel then $b_1$ is parallel with the flywheel.

# Electrical Models

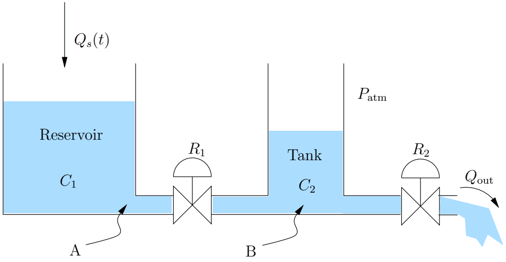


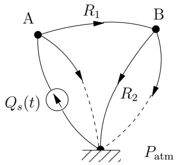

(a) Electrical model



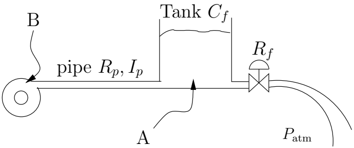

(b) Linear graph

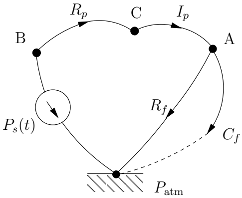
# Fluid System Models




- the pressure drop across valve $R_1$ is $P_A - P_B$ and so it is inserted between the two nodes $A$ and $B$
- the outlet valve $R_2$ discharges between the storage tank pressure $P_A$ and the reference pressure $P_{\text{atm}}$, and so it is connected in parallel with $C_2$.




the pipe is assumed to:

- dissipate energy through frictional losses at the walls
- to store energy associated with the motion of the fluid within the pipe
- they can be approximated by a combination of a single lumped resistance $R_p$ and a fluid inductnace $I_p$.

The two elements have a common flow $Q$ and are described by the elemental equation:

$$P_{R_P} = R_P Q \qquad \text{for the resistance}$$

$$P_{I_P} = I_P \frac{dQ}{dt} \qquad \text{for the iductance}$$

# Reference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