INC 693, 481 Dynamics System and Modelling: Introduction to Modelling

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Outline

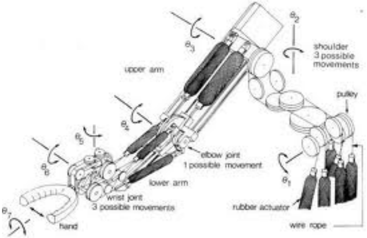
- Mathematic models and methods
- Important of Models
- Class of Models
- How to build mathematic models
- Physical modell

Approaches for solving technical control problems

1. Follow the tradition
   - follow the previous works, rely on rules of thumb
   - copy & paste
2. By experiment and trial-and-error (engineering method)
   - Build prototypes
   - Trial-and-error until your boss asks to stop!
3. Mathematical models and methods
   - Design you system by using software
   - Build a mathematical model
   - Simulate and analyse results

Example: Modeling of Humanoid Robot Dynamics

(a) Physical object



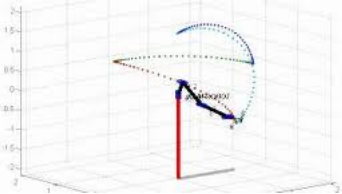
(b) Abstraction

$$g_{st}(\theta) = \begin{bmatrix} R(\theta) & p(\theta) \\ 0 & 1 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3) & -\sin(\theta_1+\theta_2+\theta_3) & 0 \\ \sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p(\theta) = \begin{bmatrix} -l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) \\ l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) \\ l_0+\theta_4 \end{bmatrix}.$$

(c) Mathematical model



(d) Simulation experiments

Mathematic models and methods

Benefits

1. Do not require a physical system
    - Can test new designs/technologies without prototype
    - Do not disturb operation of existing system
2. Can be worked with easier than the real system
    - Easy to evaluate many approaches, parameter values, ...
    - Flexible to time-scales (Mostly simulation is faster than real experiment)
    - Can access unmeasurable quantities
3. Support safety
    - Experiments mostly are dangerous
    - Operators need to be trained for such experiments
4. Help to gain insight and understanding
5. Of course, we need accurate models!

Motivation

- Economic and Profitability Requirements
- Quality and Performance Objectives
- Safety
- Environmental Requirements
- Regulations
- New Technologies - New Opportunities

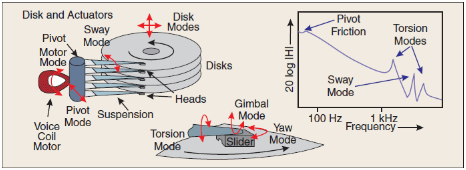
The Important of Models

Simulation: Study the system outputs for the given input

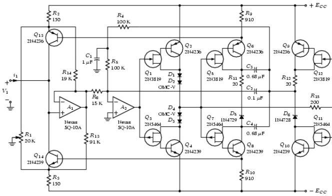
(a) Chemical Plant

(b) Thermal Study

Design an electrical, mechanical or chemical installations



(a) Harddisk Drive



(b) Electrical Circuit

The Important of Models

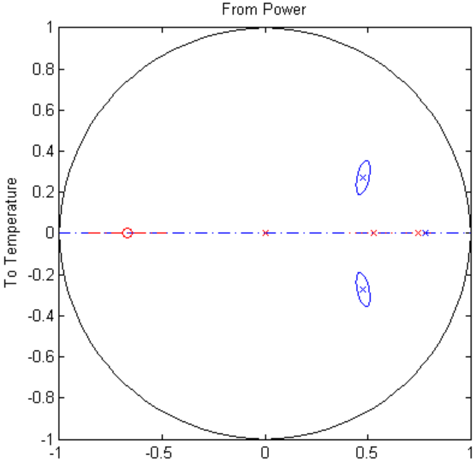
Prediction: Forcast the future values of the output

Weather forecasting, Flood forecasting

The Important of Models

Control: Model-based controller design

Pole placement controller design for tracking and disturbance rejection



The systems concepts

A way of structuring problems

- what belongs to the system, and what does not
- inputs, outputs and internal dynamics

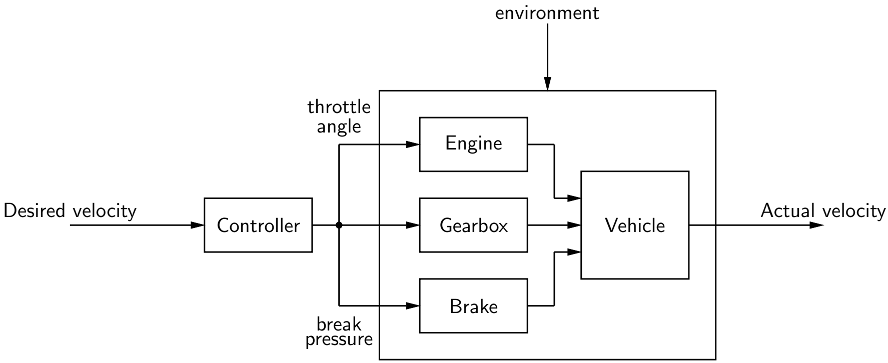


Figure: Systems view of cruise control in car

Modelling of dynamic systems

- A model M for a system S and an experiment E is anything to which E can be applied in order to answer questions about S
- A model is a tool which we use to avoid making real experiments

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\begin{aligned}
m\dot{v} &= F_{acc} + \ldots \\
J_I\ddot{\theta} &= M_f i_f - b_f\dot{\theta} + \ldots \\
C\dot{u}_b &= i_b + \ldots
\end{aligned}
$$

$\hat{y}$

Classes of Models

Many classes of models

- a piece of hardware, mental model, mathematical model, ...

We will only consider *mathematic models*

- algebraic equations, ODEs, PDEs, finite automata, ...

In this cause, we will focus on models based on differential and algebraic equations

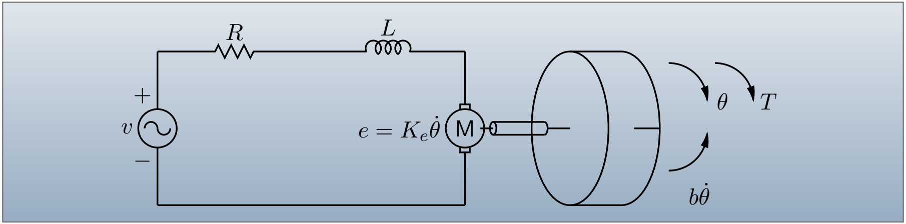
$$\dot{x}(t) = f(x(t), u(t), w(t))$$
$$0 = g(x(t), u(t), w(t))$$

How to build mathematic models?

Two basic approaches

- Physical modeling
  - First principles law
  - The Newton's law
  - The law of conservation of energy
- System Identification
  - Use experiments and observations to deduce model
  - Need prototype or real system to collect data!

Physical modeling



Assume $T = Ki,\ e = K\dot{\theta}$

Based on Newton's law and KVL

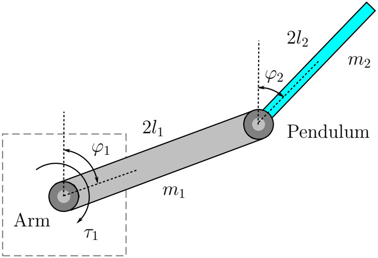
$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = v - K\dot{\theta}$$

$$\frac{\Theta}{V} = \frac{K}{s((Js+b)(Ls+R)+K^2)}$$

Physical modeling cont.

Arm-driven inverted pendulum, Kajiwara, H. et al. 1999



The motion equation:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau$$

$$(M_1 - M_2)\ddot{\varphi}_1 + R\cos(\varphi_1 - \varphi_2)\ddot{\varphi}_2$$
$$R\sin(\varphi_1 - \varphi_2)\dot{\varphi}_2^2 + (m_1 + 2m_2)l_1 g\sin(\varphi_1)$$
$$- m_2 l_2 g\sin(\varphi_2) = \tau_1$$

$$R\cos(\varphi_1 - \varphi_2)\ddot{\varphi}_1 + M_2\ddot{\varphi}_2 - R\sin(\varphi_1 - \varphi_2)\dot{\varphi}_1^2$$
$$- m_2 l_2 g\sin(\varphi_2) = 0$$

$$M_1 = \frac{4}{3}m_1 l_1^2 + \frac{4}{3}m_2 l_2^2 + 4m_2 l_1^2$$
$$M_2 = \frac{4}{3}m_2 l_2^2, \qquad R = 2m_2 l_1 l_2$$

All models are approximate

A model captures only some aspects of a system

- It is important to know which aspects are modelled and which are not
- To make sure that model is valid for intended purpose

Include everything in the model often a bad idea

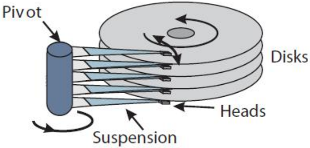
- Large and complex – hard to inspect insight
- Cumbersome and slow to manipulate
- Difficult to estimate from data (too many parameters)

Good models are simple, capture the essential behavior of the physical system.

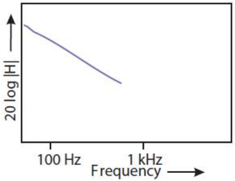
Model complexity depends on the objective

Example (Disk drive control):

The first generation controllers based on simple model



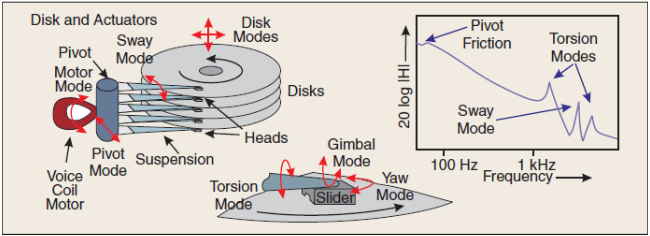
$$J_m \frac{d^2}{dt^2}\theta = M_m$$



Model complexity depends on the objective

Example (Disk drive control): Need to model more phenomena as performance demands increase

- friction, resonances, saturations, nonlinearity, etc.



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