Lecture 10: Input Disturbance

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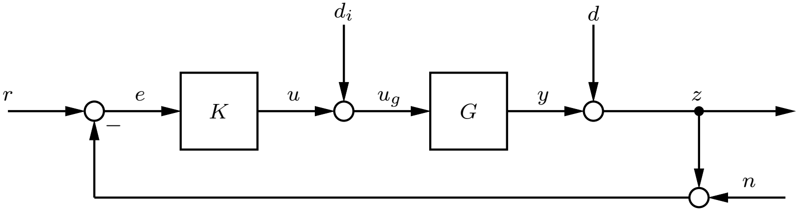
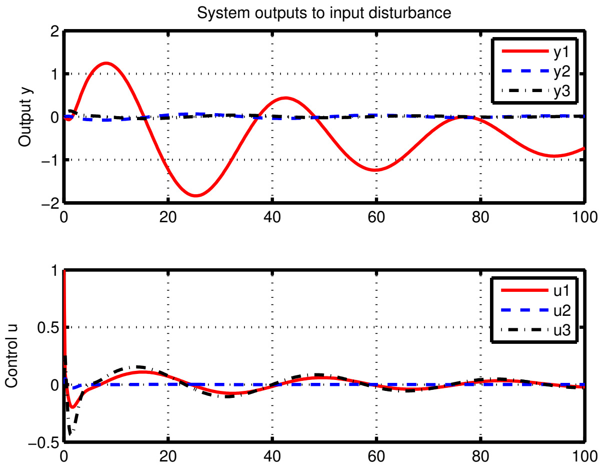
Input Disturbance



- The sensitivity S is the transfer function from an output disturbance to the controlled output.
- when $|S|$ is made small at low frequencies, the closed-loop system has a built-in capability of good rejection of output disturbances.
- this is not guaranteed for disturbances occurring at the plant input.
- from the previous Design 4, even the response to a reference step is excellent, the closed-loop system shows poor rejection of input disturbances.

Input Disturbance



Input Disturbance

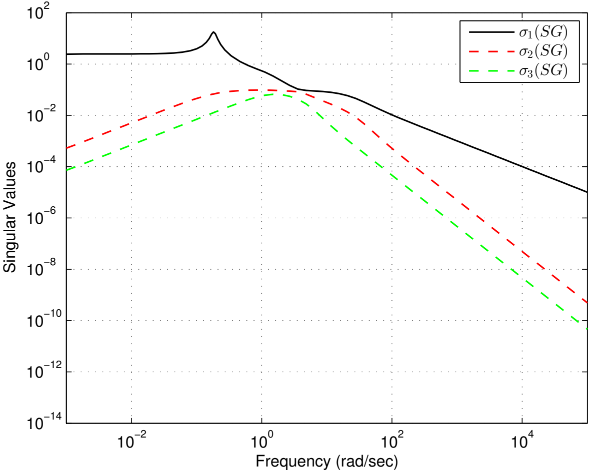
- input disturbances are likely to occur
- in the present example, they could be caused by hysteresis and friction at the aircraft control surfaces.
- the design needs to be modified to improve its disturbance rejection.
- Consider the transfer function from the input disturbance d to the controlled output y. We have (assuming $r = 0$)

$$y = G(d - Ky) \quad \text{thus} \quad (I + GK)y = Gd$$

and

$$y = (I + GK)^{-1}Gd = SGd$$

Input Disturbance



Input Disturbance

- the previous plot shows the singular values of the transfer function SG – it clearly displays the resonant peak responsible for the poor response.
- the problem is that the pole-zero cancellations are not visible when examining the transfer function $S = (I + GK)^{-1}$ and $T = (I + GK)^{-1}GK$, but are visible in $SG = (I + GK)^{-1}G$.
- Considering the SISO version of this problem

$$G(s) = \frac{n_g(s)}{d_g(s)}, \qquad K(s) = \frac{n_k(s)}{d_k(s)}$$

we have

$$SG = \frac{G}{1 + GK} = \frac{n_g d_k}{d_g d_k + n_g n_k}$$

Input Disturbance

- a pole-zero cancellation means that the polynomials d_g and n_k have a common factor, so we can write

$$d_g = \bar{d}_g \tilde{d}_g \quad \text{and} \quad n_k = \bar{n}_k \tilde{d}_g$$

where $\tilde{d}_g$ contains the plant poles that are cancelled by controller zeros.

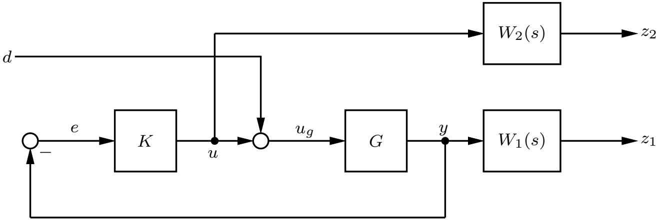
- we have

$$SG = \frac{n_g d_k}{\tilde{d}_g(\bar{d}_g d_k + n_g \bar{n}_k)}$$

which shows that the cancelled plant poles turn up as poles of the transfer function from d to y.

Input Disturbance

- to prevent such undersirable pole-zero cancellations, we can shape the transfer function SG.



- the weighting filter W_1 shapes SG, and the second filter W_2 shapes the transfer function from d to the controller output u.
- the transfer function from d to u is

$$T_K = -(I + KG)^{-1} KG$$

Input Disturbance

- this function has the same structure as the complementary sensitivity, only the loop gain GK is replaced by KG.
- Defining $z = \begin{bmatrix} z_1^T & z_2^T \end{bmatrix}^T$, we can compute the controller that minimizes the $\mathcal{H}_\infty$ norm of the closed-loop transfer function from d to z.
- with the choices

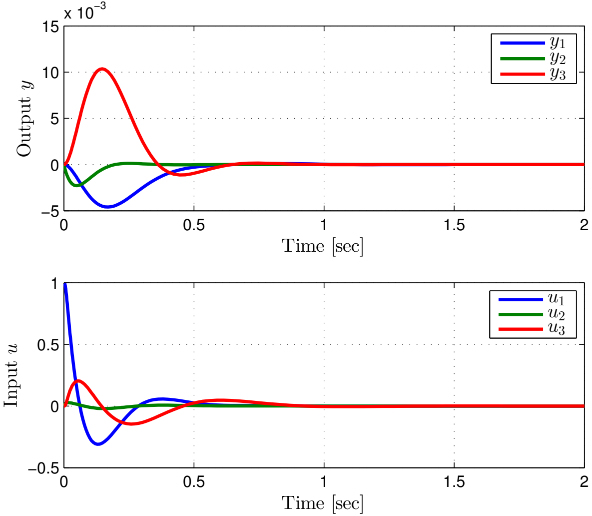
$$w_1(s) = \frac{\omega_1/M_1}{s+\omega_1} \quad \text{and} \quad w_2(s) = \frac{c}{M_2}\frac{s+\omega_2}{s+c\omega_2}$$

  where we fix $c = 10^3$.
- the new design is

ω_1	M_1	ω_2	M_2
10^{-3}	10^{-7}	5	0.25

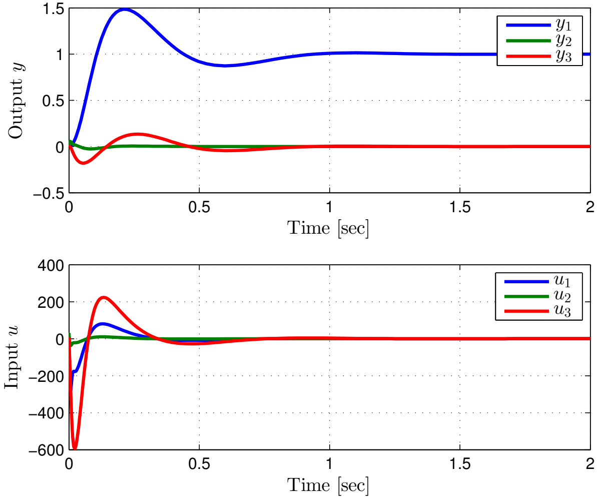
Input Disturbance

response to input disturbance $d(t) = \begin{bmatrix} \sigma(t) & 0 & 0 \end{bmatrix}^T$



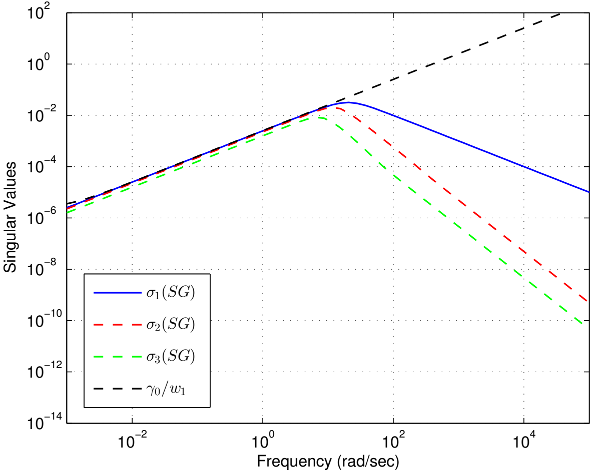
Input Disturbance

response to $r(t) = \begin{bmatrix} \sigma(t) & 0 & 0 \end{bmatrix}^T$



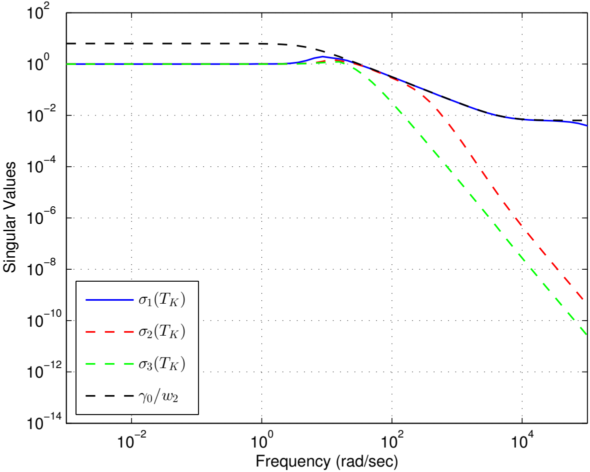
Input Disturbance

Singular values of SG and scaled constraint



Input Disturbance

Singular values of T_K and scaled constraint



Input Disturbance

- the plots show that the constraint on SG is active at low frequencies and the constraint on T_K at high frequencies.
- Since the reference input r has not been considered in the design, we would expect the tracking performance to be inferior to that in Design 4.
- it is confirmed by the response to a reference step input.
- We can shape the sensitivity S by shaping the sensitivity SG because S is contained a a factor in SG.

Reference

1. Herbert Werner "Lecture note on *Optimal and Robust Control*", 2012