INC 692: The Overview of Optimal and Robust Control

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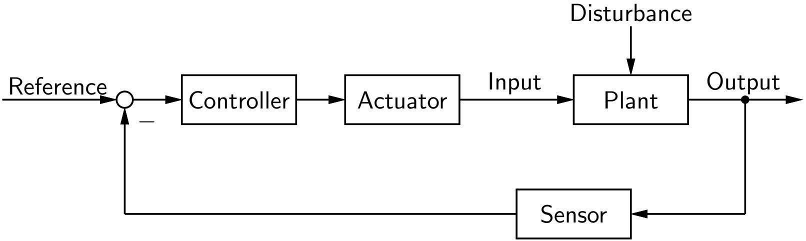
Systematic Control Design Process



1. Modeling a mathematic model
2. Analysis
3. Design a controller
4. Implementation

each step can be repeated.

Classical control course

- Modeling as a transfer function
  - Laplace transform
  - Mechanical, electrical, electromechanical systems
- Analysis
  - Time response, frequency response
  - Stability: Routh-Hurwitz criterion, Nyquist criterion
- Design
  - Root locus technique, frequency response technique
  - PID control, lead/lag compensator
- MATLAB simulation, laboratory experiments

Classical control course

Classical control in the 1930's and 1940's Bode, Nyquist, Nichols,...

- Feedback (speaker) amplifier design, Single Input Single Output (SISO)
- Frequency domain, **Graphical techniques**, trials and errors
- Emphasized design tradeoffs
  - Effects of uncertainty
  - Nonminimum phase systems
  - Performance vs. Robustness
- Problems with classical control – Overwhelmed by complex systems
  - Highly coupled multiple input, multiple output systems
  - Nonlinear systems
  - Time-domain performance specifications

State-space control course

- Modeling as a state-space model
  - Differential or difference equation
  - Linear algebra
- Analysis
  - Stability, controllability, observability
  - Realization, minimality
- Design
  - State feedback, observer
  - LQR, LQG
- MATLAB simulation, laboratory experiments

Early Year:

- Wiener (1930's - 1950's) Generalized harmonic analysis, cybernetics, filtering , prediction, smoothing
- Kolmogorov (1940's) Stochastic processes
- Linear and nonlinear programming (1940's -)

Optimal Control:

- Bellman's Dynamic Programming (1950's)
- Pontryagin's Maximum Principle (1950's)
- Linear optimal control (late 1950's and 1960's)
  - Kalman Filtering
  - Linear-Quadratic (LQ) regulator problem
  - Stochastic optimal control (LQG)

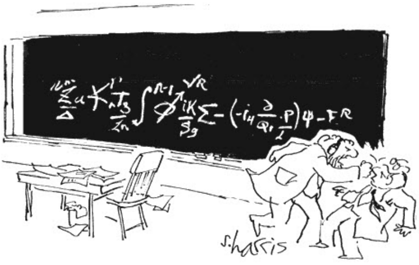
Brief history of control theory

- Classical control (-1950)
  - Transfer function
  - Frequency domain
- Modern control (1960-)
  - State-space model
  - Time domain
- Post(neo)-modern control (1980-)
  - Robust control
  - LPV control
  - etc.

Brief history of control theory

Figure 1.1: A picture history of control

Brief history of control theory



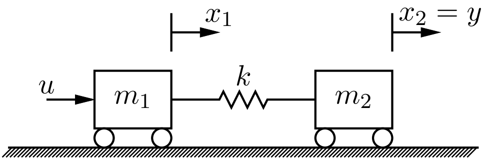
"You want proof? I'll give you proof!"

The true story of Post Modern Control System Engineers.

- We are not mad but want proofs.
- Improve performance of 5-10% would be the great success.

Motivation: ACC Benchmark problem

Two-cart (frictionless) system



Differential equations

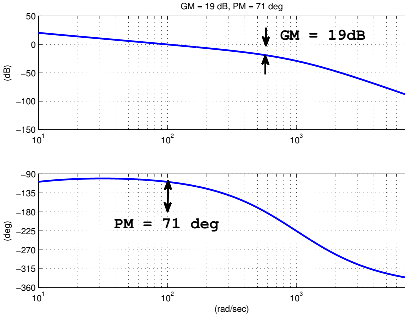
$$m_1\ddot{x}_1 = u(t) - k(x_1(t) - y(t))$$
$$m_2\ddot{y} = -k(y(t) - x_1(t))$$

Transfer function

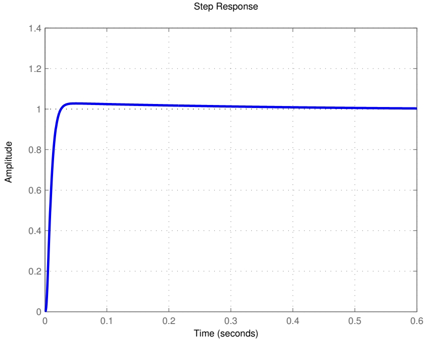
$$P(s) = \frac{y(s)}{u(s)} = \frac{k}{s^2(m_1 m_2 s^2 + k(m_1 + m_2))}$$

ACC Benchmark problem

Open-loop frequency response



Closed-loop step response



- New assumption
  - Mass and spring constant are uncertain

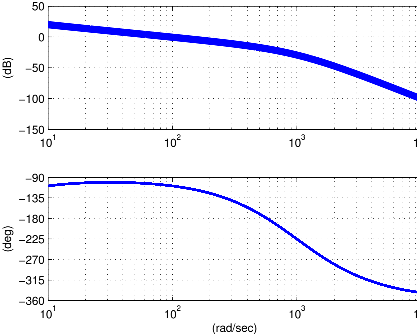
$$k, m_1, m_2 \in [0.8, 1.2]$$

$$P(s) = \frac{k}{s^2(m_1 m_2 s^2 + k(m_1 + m_2))}$$

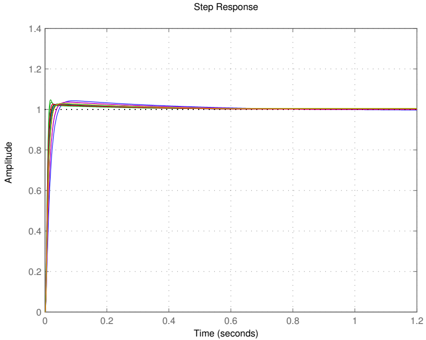
- For perturbations of these parameters, how will the CL stability and performance change?

ACC Benchmark problem
Monte Carlo (random) sampling

Open-loop frequency response



Closed-loop step response



Luckily it seems robust.

ACC Benchmark problem

- New assumption
  - Mass and spring constant are uncertain

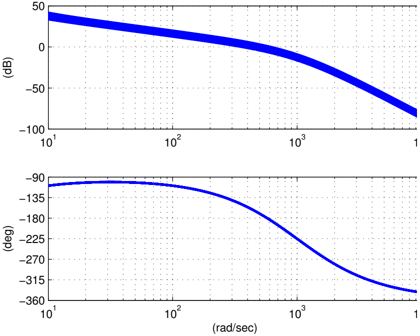
$$k = 6 \pm 20\%, m_1, m_2 \in [0.8, 1.2]$$

$$P(s) = \frac{k}{s^2(m_1 m_2 s^2 + k(m_1 + m_2))}$$

- For perturbations of these parameters, how will the CL stability and performance change?

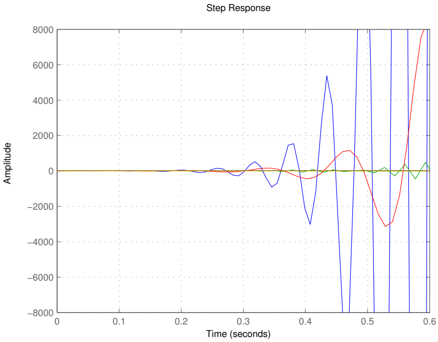
ACC Benchmark problem

Open-loop frequency response



Not robustly stable!

Closed-loop step response



Some thoughts

- By Monte Carlo sampling, we can never be 100% sure that the closed-loop system is stable and performs well for any perturbation.
- Without random sampling, can we know if perturbed systems are always stable and have satisfactory performance for any perturbation?
- What is the "smallest" perturbation to violate stability and performance spaces?
- What are the worst-case perturbation and performance?
- For specified uncertainty and performance specs, can we design a controller which works robustly?

Real life

$$F - F_{\text{friction}} = m\ddot{x}$$

- Mass m varies between 100 ton to 250 ton
- Friction force F_{friction} is an unknown disturbance

- How to suppress the disturbance?
- How to design a robust controller, i.e. a controller that achieves stability and good performance for all values of m?

MIMO System Example

Consider a multi-input/multi-output (MIMO) system:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = G(s) \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \quad \text{with} \quad G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{4}{s+8} \\ \frac{0.5}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

Suppose we neglect off-diagonal terms and choose the control structure

$U_1(s) = K_1(s)(R_1(s) - Y_1(s))$ and $U_2(s) = K_2(s)(R_2(s) - Y_2(s))$, i.e.,

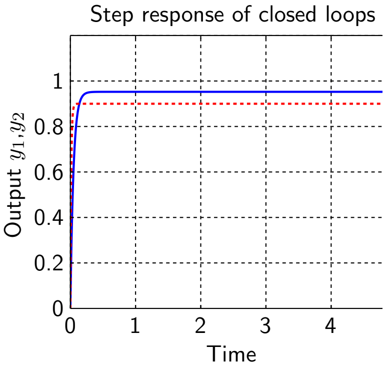
$$\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} = K(s) \begin{bmatrix} R_1(s) - Y_1(s) \\ R_2(s) - Y_2(s) \end{bmatrix} \text{ with } K(s) = \begin{bmatrix} K_1(s) & 0 \\ 0 & K_2(s) \end{bmatrix}$$

with $K_1(s) = 20$ and $K_2(s) = 18$.

MIMO System Example

K_1 gives nice response for G_{11}

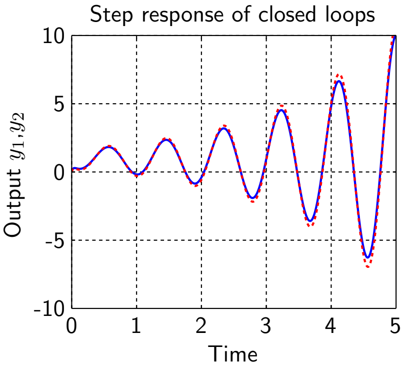
K_2 gives nice response for G_{22}



$$G_{c1} = G_{11}K_1/(1 + G_{11}K_1)$$

$$G_{c2} = G_{22}K_2/(1 + G_{22}K_2)$$

But: overall closed loop is unstable



$$G_c = GK(I + GK)^{-1}$$

MIMO System Example

Why is the closed loop unstable?

ans: G has zero in right half-plane, and controllers gains are too big

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{4}{s+8} \\ \frac{0.5}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

- How can we determine properties of MIMO systems?
- How should we design MIMO controllers for MIMO plants?

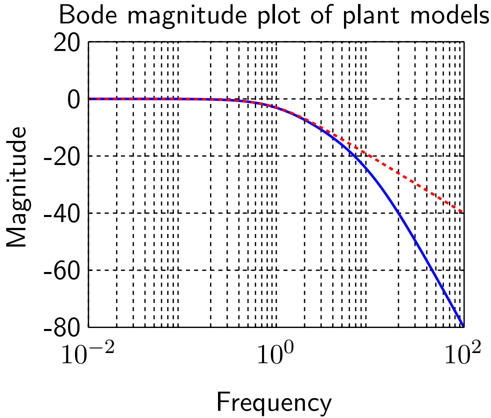
MIMO System Example 2

Real system:

$$G_{\text{real}}(s) = \frac{1}{s+1} \cdot \frac{1}{(0.1s+1)^2}$$

Approximation:

$$G(s) = \frac{1}{s+1}$$



Bode plots are similar, differences only for large frequencies. Place poles far from stability border in left half-plane to be on safe side.

MIMO System Example 2

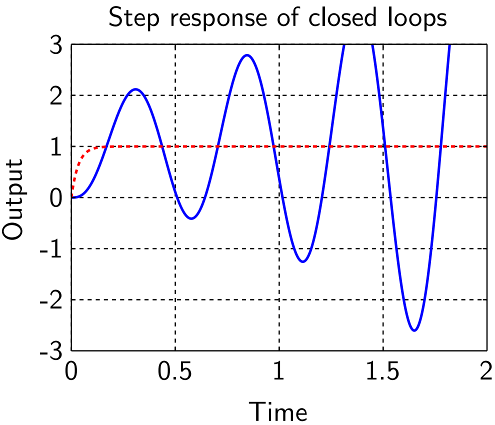
Controller:

$$K_(s) = \frac{30(s+1)}{s}$$

achieves

$$G_{cl}(s) = \frac{GK}{1+GK} = \frac{30}{s+30}$$

i.e. pole at $s = -30$
but simulation of real
closed loop shows
instability!

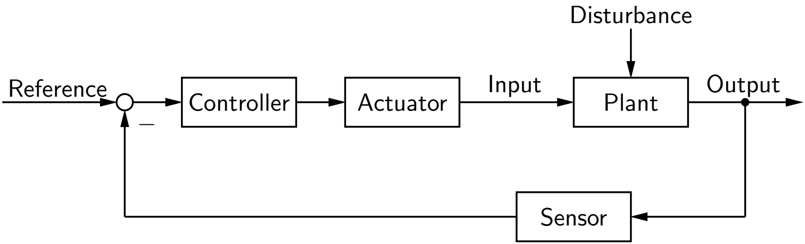


- What means "good robustness"   "to be on safe side"?
- How to analyze robustness?
- How to achieve robustness systematically?

Why is robust control important?

- A model is never precise!
  - Difficulty in identifying parameters and high frequency plant dynamics
  - Product variability
  - Uncertainty in disturbances, references, and measurement noise
- Without taking such uncertainties into account, we never know if the designed controller works in actual implementations.

Robust Controller Design Process



1. Modeling an uncertain model
2. Analysis
3. Design a controller
4. Implementation

each step can be repeated.

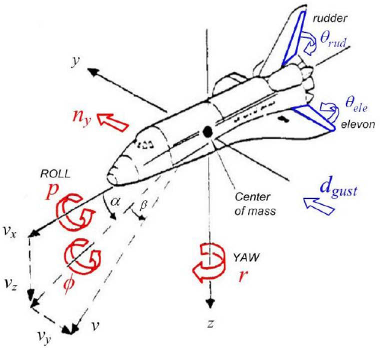
Robust Controller Design Process

Main problems

- Modeling: Uncertainty modeling
  - Build a mathematical model of uncertainties in the plant and the disturbance signals.
- Analysis: Robustness analysis
  - Given an open-loop or closed-loop system, determine if the system satisfies robust stability and/or robust performance
- Design: Robust controller design
  - Design and controller guaranteeing robust stability and/or robust performance.

- A system (open-loop or closed-loop)
  - is *nominally stable (NS)* if it is stable with no model uncertainty.
  - satisfies *nominal performance (NP)* if it satisfies performance spaces with no model uncertainty.
  - is *robustly stable (RS)* if it is stable for any plant with specified model uncertainty.
  - satisfies *robust performance (RP)* if it satisfies performance specs for any plant with specified model uncertainty.

Example 2: Space Shuttle Reentry Control



$$x = \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} = \begin{bmatrix} \text{side slip angle} \\ \text{roll rate} \\ \text{yaw rate} \\ \text{bank angle} \end{bmatrix}$$

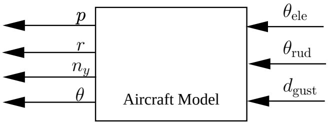
$$y = \begin{bmatrix} p \\ r \\ n_y \\ \phi \end{bmatrix}, \quad n_y = \text{lateral acceleration}$$

$$u = \begin{bmatrix} \theta_{\text{ele}} \\ \theta_{\text{rud}} \\ d_{\text{gust}} \end{bmatrix} = \begin{bmatrix} \text{elevon surface angle} \\ \text{rudder surface angle} \\ \text{lateral wind gusts} \end{bmatrix}$$

[1] Doyle, et al, "Design example using μ-synthesis: space shuttle lateral axis FCS during reentry", IEEE CDC, 1986

[2] Mu toolbox for MATLAB, user guide

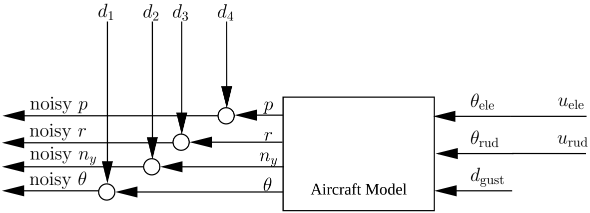
Example 2: Space Shuttle Reentry Control



Aerodynamic coefficients: relation between forces/torques and sides slip/rudder/elevon angle

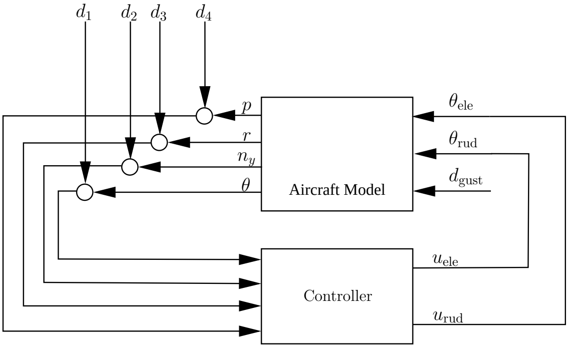
- Estimated from theoretical predications, numerical calculations, experiments in wind tunnels and/or flight tests
- Shuttle at e.g. Mach 0.9 $\longrightarrow$ mixture of subsonic and supersonic flows
- Highly uncertain coefficients

Example 2: Space Shuttle Reentry Control



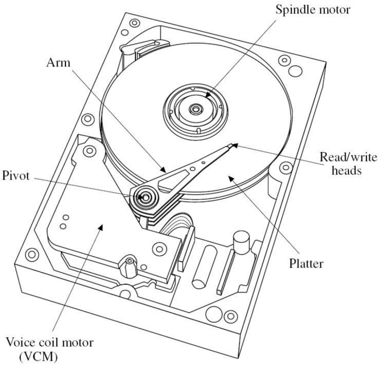
- Each measurement is corrupted by noise
- The measurement noise becomes more severe with at higher frequencies
- Actuators have dynamics and saturations
- There are possibly time delays in control commands

Example 2: Space Shuttle Reentry Control



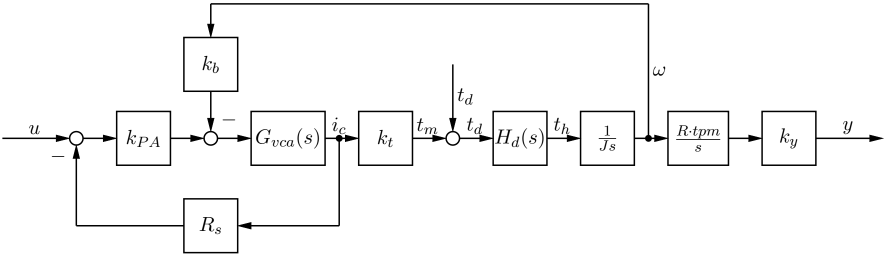
Stability? Performance? of Real Life System

Example 3: Hard Disk Drive Control



- Smaller devices with larger information density require smaller track width and lower tolerance in positioning of read/write heads
- Rotational speed of 15000 rpm., 30000 tracks per inch.
- Presence of parameter variations and uncertainties, nonlinearities and noise

Example 3: Hard Disk Drive Control

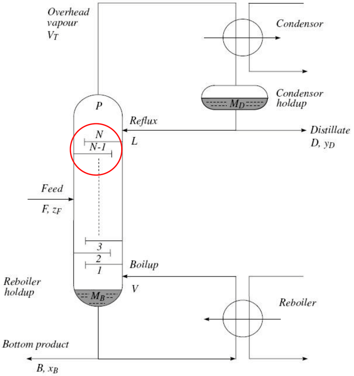


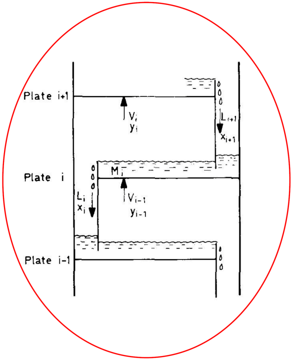
- All parameters of harddrive have bounded tolerances.
- t_d are torque disturbances
- $H_d(s)$ has uncertain resonances
- the output y is included a measurement noises

Example 4: Distillation Column

- Separation and purification of chemicals based on difference at boiling points of multicomponent liquids
- Difficult to control:
  - highly nonlinear process
  - high order models
  - linearized models often ill-conditioned
  - parametric gain uncertainties
  - time delays (up to 1 min at input)

Example 4: Distillation Column





[1] Skogestad, et al., *Robust Control of ill-conditioned plants: high purity distillation*, IEEE TAC, 1988

[2] Gu, Petkov, Konstantinov, *Robust Control Design with MATLAB*, Springer-Verlag, 2005

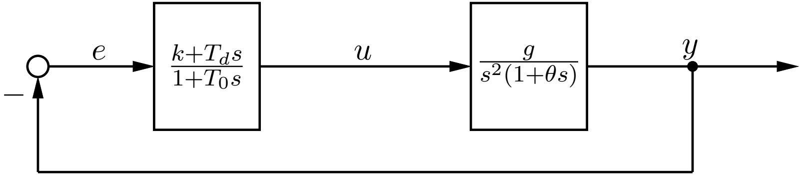
Conclusion

- **Space shuttle reentry control:** ensure robust stability (safety) and trajectory tracking in spite of complex physics (subsonic and supersonic aerodynamics),
- **Harddrive control:** pushing the performance of the devices to the limits, while accounting for tolerances in series device production
- **Distillation column:** increase efficiency of high order nonlinear system

Sources of uncertainty:

- Signal uncertainty:
  - noise, exogenous disturbances;
  - unknown reference tracking commands.
- Dynamic uncertainty:
  - unmodeled dynamics;
  - parametric variations;
  - operating point changes;
  - nonlinearities not captured by the linear model;
  - non-repeatable system behavior.
- Dynamic uncertainty is potentially destabilizing under feedback
- Robust control techniques: deal with uncertainty in a systematic way. We have to model the uncertainties.

Parametric Uncertainty



$$G(s) = \frac{g}{s^2(1+s\theta)}, \qquad g \in [g_1, g_2], \quad \theta \in [\theta_1, \theta_2]$$

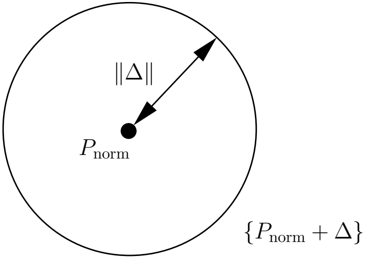
$$g = g_0 + \frac{g_2 - g_1}{2}\delta_g, \qquad g_0 = \frac{g_1 + g_2}{2}, \qquad |\delta_g| \le 1$$

$$\theta = \theta_0 + \frac{\theta_2 - \theta_1}{2}\delta_\theta, \qquad \theta_0 = \frac{\theta_1 + \theta_2}{2}, \qquad |\delta_\theta| \le 1$$

Nominal system: $G_0(s) = \dfrac{g_0}{s^2(1+s\theta_0)}$

A set of **perturbed systems:** $G(s) = \dfrac{g(\delta)}{s^2(1+s\theta(\delta))}$

Robust control models: set descriptions



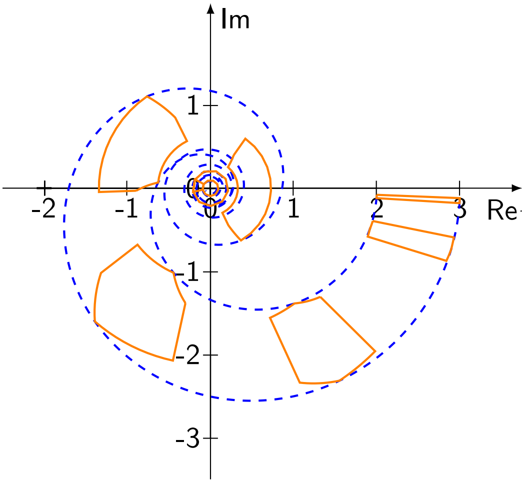
The perturbations, Δ, are specified by a class

$$\mathcal{X}_\Delta := \{\Delta | \Delta \text{ is causal, stable, LTI } \}$$

Typical (additive) robust control models:

$$\mathcal{P} := \{P_{\text{norm}} + \Delta | \Delta \in \mathcal{X}_\Delta, \|\Delta\| \leq 1\}.$$

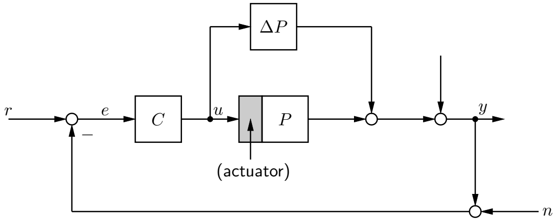
Example



$$P(s) = \frac{k}{1+\tau s} e^{-\theta} s,$$

$$2 \leq k, \theta, \tau \leq 3$$

$$\omega = 7, 2, 1, 0.5, 0.2, 0.05, 0.01$$



Uncertainties

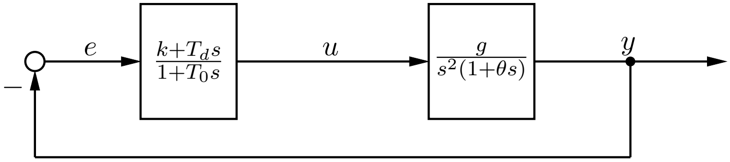
- Unmodelled dynamics
- Time variance
- varying loads
- Manufacturing variance
- Limited identification
- Actuators and sensors

Goals:

- Stability
- Tracking
- Disturbance rejection
- Sensor noise rejection

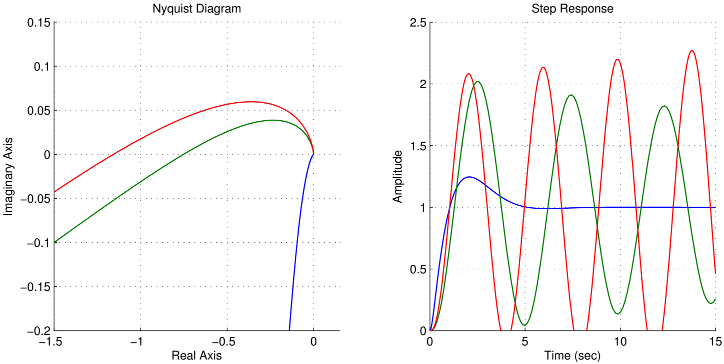
Robustness

Parametric Uncertainty



- Nominal plat: $g = g_0 = 1, \quad \theta = 0$
- Controller : $k = 1, T_d = \sqrt{2}, \quad T_0 = 1/10$
- Perturbed plant: $g \in [\underline{g}, \overline{g}], \quad \theta \in [0, \overline{\theta}]$

Parametric Uncertainty

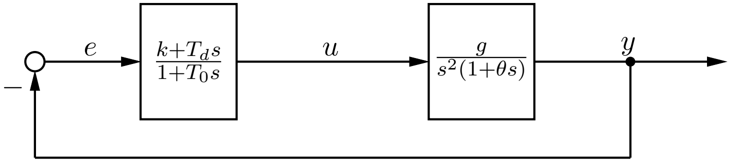


Blue: $g_0 = 1 \qquad \theta_0 = 0$

Green: $g_0 = 1.3 \qquad \theta_0 = 1$

Red: $g_0 = 2 \qquad \theta_0 = 1$

Parametric Uncertainty

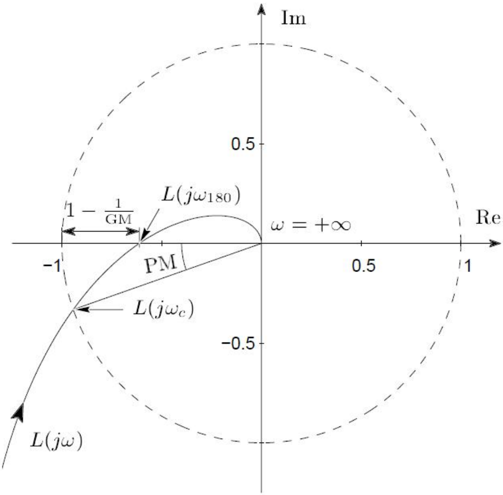


Closed loop characteristic polynomial:

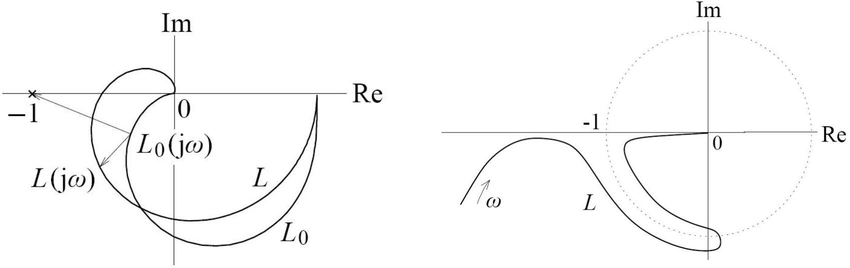
$\mathcal{X}(s) = \theta T_0 s^4 + (\theta + T_0)s^3 + s^2 + gT_d s + gk$ stable for all g and θ?

- Routh-Hurwitz criterion, Kharitonov, Edge theorem, etc.
- Synthesis? Closed-loop performance?

Gain and Phase Margins (Nyquist Plot)



Gain and Phase Margins (Nyquist Plot)



Nominal and perturbed Nyquist plots

Control Systems

Classical Control

- Rules of thumb, gain and phase margins
- All allowed perturbations on dynamics were not quantized
- Only stability margins are guarded, not performance
- Mostly for SISO systems
- For MIMO systms: LQR, LQG

Robust Control

- Provide strict and well defined criteria
- Clear descriptions for the allowed perturbations
- Not only stability but also performance robustness
- MIMO systems (ready to use tools, $\mathcal{H}_\infty, \mu$)

Reference

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5. J. C. Doyle, B. A. Francis and A. R. Tannenbaum, *Feedback Control Theory*, McMillan, 1992.
6. R. Nagamune, *A Lecture note on Multivariable feedback control*