INC 691 Linear and Nonlinear System Identification **Homework 9**

Assign: Fri 18 May 2012 **Due: Tue 29 May 2012**

Instruction: Hand in your work in the mail box labeled INC691 by 4 pm. or submit it via email. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given zero for the homework.

1. Show that for a quadratic cost function $V(\theta)$, the steepest descent method according to $\theta(l+1) = \theta(l) - \alpha g(l)$ will be stable if the learning rate satisfies

$$\alpha < \frac{2}{\lambda_{\max}(\nabla^2 V)},$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a symmetric matrix.

Solution:

$$\begin{aligned} F(x) &= \frac{1}{2}x^T Q x + p^T x + r \\ \nabla F(x) &= Qx + p \end{aligned}$$

From the discrete-time linear system

$$\begin{aligned} x(l+1) &= x(l) - \alpha g(l) = x(l) - \alpha(Qx(l) + p) \\ &= [I - \alpha Q]\, x(k) - \alpha p. \end{aligned}$$

The convergence of the learning rate can be tested by checking the eigenvalues of the matrix $[I - \alpha Q]$.

$$[I - \alpha Q]\, z_i = z_i - \alpha Q z_i = z_i - \alpha \lambda_i z_i = (1 - \alpha\lambda_i) z_i$$

The stability criteria is

$$\begin{aligned} |(1 - \alpha\lambda_i)| < 1 \qquad &\Longrightarrow \qquad -1 < 1 - \alpha\lambda_i < 1 \\ \alpha > 0 \qquad &\text{or} \qquad \alpha < \frac{2}{\lambda_i} \\ \alpha < \frac{2}{\lambda_{max}(Q)} &= \frac{2}{\lambda_{max}(\nabla^2 F(x))} \end{aligned}$$