INC 691 Linear and Nonlinear System Identification **Homework 6**
Assign: Fri 11 May 2012 **Due: Fri 18 May 2012**

Instruction: Hand in your work in the mail box labeled INC691 by 4 pm. or submit it via email. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given zero for the homework.

1. Show that the eigenvalues of the Hessian of a quadratic function are equal to the second derivatives of that function in the direction of the corresponding Hessian eigenvectors.
   Solution:
   Let $F(x) = \frac{1}{2}x^TQx + p^Tx + r$ and assume that ξ are eigenvectors of the Hessian of $F(x)$. The second derivatives of the function in the direction of the eigenvectors is

$$\nabla_\xi F(x) = \frac{\xi^T \nabla^2 F(x) \xi}{\|\xi\|^2} = \frac{\xi^T Q \xi}{\|\xi\|^2}. \tag{1}$$

   The eigenvalues of the Hessian of the quadratic function can be computed by

$$\begin{aligned} Q\xi &= \xi\Lambda \\ \xi^T Q \xi &= \xi^T \xi \Lambda \\ \Lambda &= \frac{\xi^T Q \xi}{\|\xi\|^2}, \end{aligned}$$

   which is equivalent to (1).

2. Let $V : \mathbb{R}^2 \to \mathbb{R}$, $V(x) = x_1^2 - 2x_1 + 3x_1x_2^2 + 4x_2^3$, $x_0 = \begin{bmatrix}1 & 1\end{bmatrix}^T$, $f = \begin{bmatrix}-2, 1\end{bmatrix}^T$. Find the first and second order of the directional derivative of $V(x)$ along f.
   Solution: Do it by yourself.