INC 691 Linear and Nonlinear System Identification **Homework 5**
Assign: Fri 10 Feb 2012 **Wed 15 Feb 2012**

Instruction: Hand in your work in the mail box labeled INC691 by 4 pm. or submit it via email. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given zero for the homework.

1. Given a true model

$$y(k) = \frac{z^{-3}(0.103 + 0.181z^{-1})}{1 - 1.991z^{-1} + 2.203z^{-2} - 1.841z^{-3} + 0.894z^{-4}}u(k) + \frac{1}{1 - 1.991z^{-1} + 2.203z^{-2} - 1.841z^{-3} + 0.894z^{-4}}e(k)$$

and chosen the model structure $\mathcal{M}$

$$\mathcal{M} = \left\{ G(z,\theta) = \frac{z^{-3}(b_0 + b_1 z^{-1})}{1 + f_1 z^{-1} + f_2 z^{-2} + f_3 z^{-3} + f_4 z^{-4}}; \quad H(z,\theta) = \frac{1}{1 + f_1 z^{-1} + f_2 z^{-2} + f_3 z^{-3} + f_4 z^{-4}} \right\}.$$

Apply 30 times the same sequence $u(k) \in \mathcal{N}(0,2)$ of length $N = 2000$ and measure the corresponding $y(k)$, where $e(k)$ is a white noise with $\mathcal{N}(0, 0.02)$. Assume that we do not know the true model, use PEM to approximate parameters

$$\hat{\theta}_N = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 & b_0 & b_1 \end{bmatrix}.$$

- Determine the mean bias error of your approximation as

$$\text{bias error } = \left\| \theta_0 - \mathrm{E}\left[\hat{\theta}_N\right] \right\|$$

- Determine the variance of your approximation as

$$\text{variance } = \left\| \mathrm{E}\left[\left(\hat{\theta}_N - \mathrm{E}\left[\hat{\theta}_N\right] \right)^2 \right] \right\|$$

- Discuss about your results.