Lecture 3a: Block Diagram

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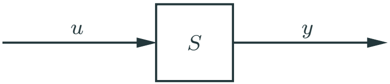
Block diagrams

systems often denoted by **block diagram**:



- lines with arrow denote signals
- boxes denotes systems: arrows show inputs and outputs
- special symbols for some systems

Block diagrams

scaling system: $y(t) = au(t)$

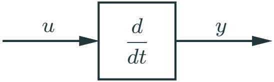
- called an *amplifier* if $|a| > 1$
- called an *attenuator* if $|a| < 1$
- called *inverting* if $a < 0$
- a is called the *gain* or *scaling factor*

Usually, denoted by triangle or rectangle in block diagram:

Block diagrams

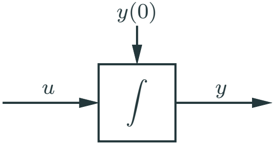
differentiator: $y(t) = \frac{du}{dt}$

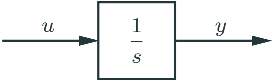
commonly used notations for differentiator:



integrator: $y(t) = \int_a^t u(\tau) d\tau$ (a is often 0 or $-\infty$)

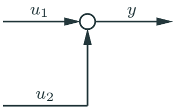
commonly used notations for integrator:



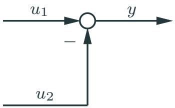


Example with multiple inputs

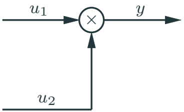
- summing system: $y(t) = u_1(t) + u_2(t)$



- difference system: $y(t) = u_1(t) - u_2(t)$



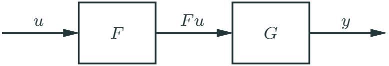
- multiplier system: $y(t) = u_1(t)u_2(t)$



Interconnections of systems

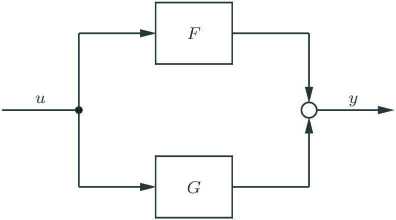
we can interconnect systems to form new systems, e.g.,

- cascade (or series): $y = G(Fu) = GFu$



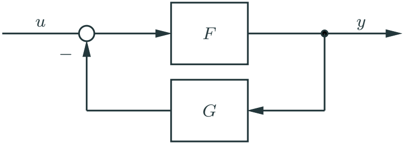
  (note the block diagrams and algebra are reversed)

- sum (or parralled): $y = Fu + Gu$



Interconnections of systems

- Feedback: $y = F(u - Gy)$

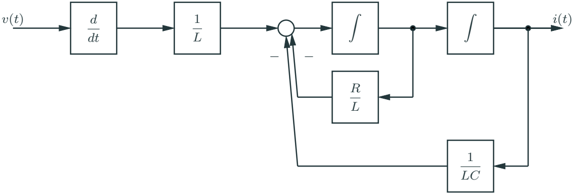


  - the minus sign is for a negative feedback while the plus sign is for a positive feedback.

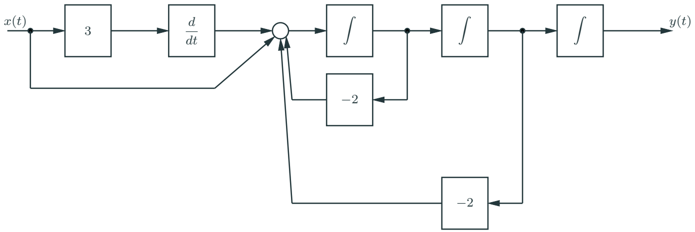
Block Diagram

$$L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = v(t)$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i(t) = \frac{1}{L}\frac{dv}{dt}$$



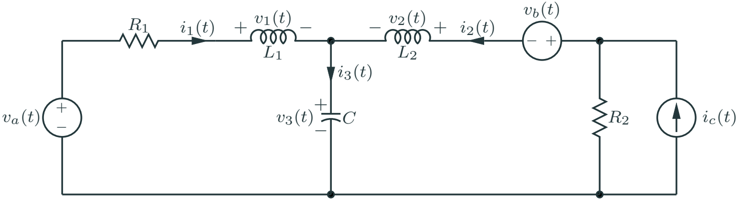
Block Diagram



$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = 3\frac{dx}{dt} + x(t)$$

Block Diagram: RLC circuit

A circuit shown below has three state equations:



$$v_1(t) = v_a(t) - R_1 i_1(t) - v_3(t)$$

$$v_3(t) = -v_2(t) - v_b(t) + R_2(i_c(t) - i_2(t))$$

$$v_2(t) = -R_2 i_2(t) - v_3(t) - v_b(t) + R_2 i_c(t)$$

$$i_3(t) = i_1(t) + i_2(t)$$

Block Diagram: RLC Circuit

Since

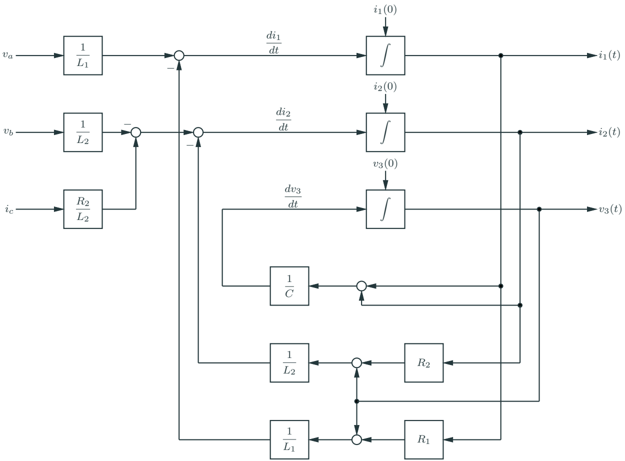
$$v_1(t) = L_1 \frac{di_1(t)}{dt},$$
$$v_2(t) = L_2 \frac{di_2(t)}{dt},$$
$$i_3(t) = C \frac{dv_3(t)}{dt},$$

we obtain the dynamic equation in state form :

$$\frac{di_1(t)}{dt} = -\frac{R_1}{L_1} i_1(t) - \frac{1}{L_1} v_3(t) + \frac{1}{L_1} v_a(t) \quad (1)$$
$$\frac{di_2(t)}{dt} = -\frac{R_2}{L_2} i_2(t) - \frac{1}{L_2} v_3(t) - \frac{1}{L_2} v_b(t) + \frac{R_2}{L_2} i_c(t) \quad (2)$$
$$\frac{dv_3(t)}{dt} = \frac{1}{C} i_1(t) + \frac{1}{C} i_2(t) \quad (3)$$

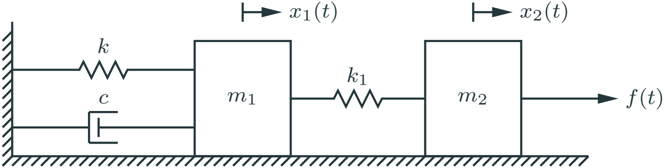
Block Diagram: RLC Circuit

Note that our required outputs are $i_1(t), i_2(t)$, and $v_3(t)$.



Block Diagram: Sprint-Mass-Damper System

There are no frictions between masses and floors.



ODE:

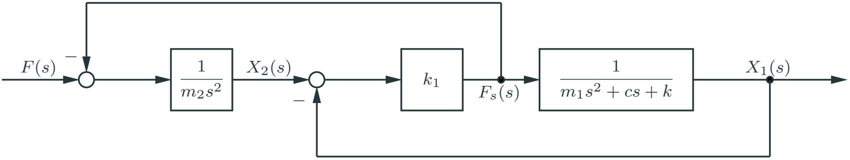
$$m_1\ddot{x}_1 + c\dot{x}_1 + (k + k_1)x_1 - k_1 x_2 = 0$$
$$m_2\ddot{x}_2 + k_1 x_2 - k_1 x_1 = f$$

Laplace Transform:

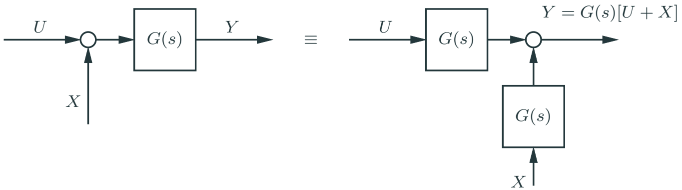
$$(m_1 s^2 + cs + k)X_1(s) = k_1(X_2(s) - X_1(s))$$
$$m_2 s^2 X_2(s) = F(x) - k_1(X_2(s) - X_1(s))$$

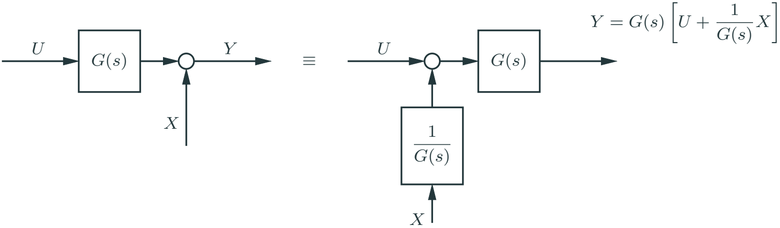
Block Diagram: Sprint-Mass-Damper System

$$X_1(s) = \frac{1}{m_1 s^2 + cs + k}\left(k_1(X_2(s) - X_1(s))\right)$$
$$X_2(s) = \frac{1}{m_2 s^2}\left(F(x) - k_1(X_2(s) - X_1(s))\right)$$



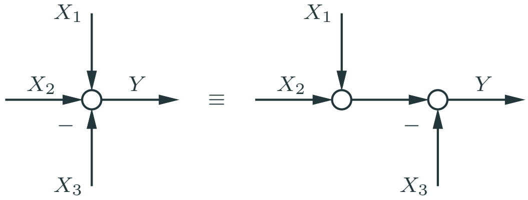
Block Diagram: Moving across a summing junction





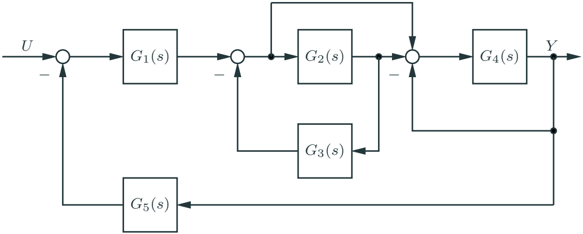
Consider

$$Y = X_1 + X_2 - X_3 = (X_1 + X_2) - X_3$$

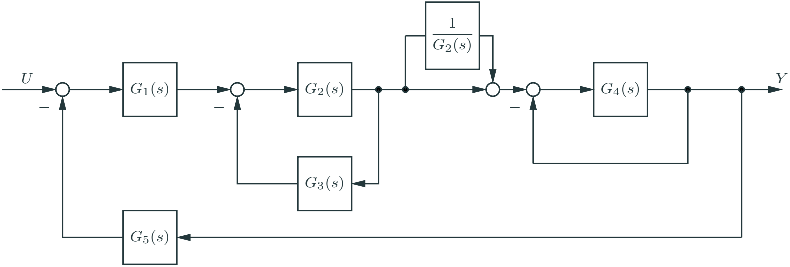


$Y = Y$

Find the transfer function $Y(s)/U(s)$ of the block diagram.



Hint: $\frac{Y(s)}{U(s)} = \frac{G_1(s)[1+G_2(s)]G_4(s)}{[1+G_2(s)G_2(s)][1+G_4(s)]+G_1(s)[1+G_2(s)]G_4(s)G_5(s)}$



Reference

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3. Lathi, B. P., *Signal Processing & Linear Systems*, Berkeley-Cambridge Press, 1998
4. Hassan, K. Khalil, *Control Systems: An Introduction*, Michigan Publishing, 2023