Lecture 2a: Time-Domain Analysis of Continuous-Time Systems with MATLAB

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Matlab

Zero-Input Response $y_0(t)$

To solve a differential equation, you can use a command **dsolve** to solve the equation.
For an continuous-time LTI system specified by the differential equation

$$(D^2 + 4D + k)y(t) = (3D + 5)f(t)$$

determine the zero-input component of the response if the initial conditions are $y_0(0) = 3$, and $\dot{y}_0(0) = -7$ for two values of k: (a) 3 (b) 4 (c) 40.

```
syms y(t) t
Dy = diff(y, 1);
cond1 = y(0) == 3; cond2 = Dy(0) == -7; conds = [cond1; cond2];

sys1 = diff(y,2) + 4*diff(y,1) + 3*y == 0
```

$$\text{sys1(t)} = \frac{\partial^2}{\partial t^2}\, y\,(t) + 4\,\frac{\partial}{\partial t}\, y\,(t) + 3\, y\,(t) = 0$$

Zero-Input Response $y_0(t)$

```
sys2 = diff(y,2) + 4*diff(y,1) + 4*y == 0
```

$$\text{sys2(t)} = \frac{\partial^2}{\partial t^2}\, y\,(t) + 4\,\frac{\partial}{\partial t}\, y\,(t) + 4\,y\,(t) = 0$$

```
sys3 = diff(y,2) + 4*diff(y,1) + 40*y == 0
```

$$\text{sys3(t)} = \frac{\partial^2}{\partial t^2}\, y\,(t) + 4\,\frac{\partial}{\partial t}\, y\,(t) + 40\,y\,(t) = 0$$

```
y1 = dsolve(sys1, conds)
```

$$\text{y1} = \mathrm{e}^{-t} + 2\,\mathrm{e}^{-3\,t}$$

Zero-Input Response $y_0(t)$

```
y2 = dsolve(sys2, conds)
```

$$\mathrm{y2} = -e^{-2\,t}\,(t-3)$$

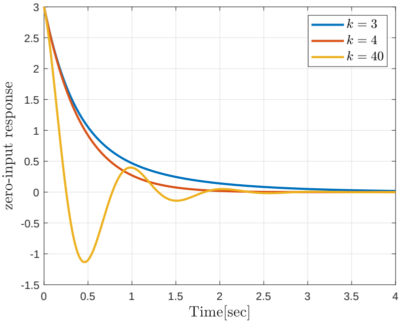
```
y3 = dsolve(sys3, conds)
```

$$\mathrm{y3} = \frac{e^{-2\,t}\,(18\,\cos(6\,t) - \sin(6\,t))}{6}$$

Zero-Input Response $y_0(t)$

To plot y_0 respect to t, we can use Matlab's code as follow:

```
ts = -1:0.01:10;
y1c = y1*heaviside(t);
y2c = y2*heaviside(t);
y3c = y3*heaviside(t);
y1p = subs(y1c,t,ts); y2p = subs(y2c,t,ts); y3p = subs(y3c,t,ts);
plot(ts,y1p,ts,y2p,ts,y3p, 'linewidth', 2)
h = legend('k =3', 'k=4', 'k=40');
```



Zero-State Response $y_s(t)$

For an LTI system specified by the differential equation

$$(D^2 + 3D + 2)y(t) = Df(t)$$

To calculate the zero-state response, we can use MATLAB to calculate as follow:

$$h(t) = b_n\delta(t) + P(D)y_n(t)\mathbb{1}(t)$$

In this case $b_n = 0$ and the initial values of $y_n(t)$ are $y_n(0^-) = 0$ and $\dot{y}(0^-) = 1$.

```
syms y(t) t

Dy = diff(y,1);
cond1 = y(0) == 0; cond2 = Dy(0) == 1;
conds = [cond1; cond2]; sys4 = diff(y,2) + 3*diff(y,1) + 2*y(t) == 0;
y_n = dsolve(sys4, conds)
```

$$\text{y\_n} = \quad e^{-t} - e^{-2\,t}$$

Zero-State Response $y_s(t)$

```
Dy_n = diff(y_n)
```

$$\text{Dy\_n} = 2\,\mathrm{e}^{-2\,t} - \mathrm{e}^{-t}$$

Therefore

$$h(t) = 0 + (2e^{-2t} - e^{-t})\mathbb{1}(t)$$

Zero-State Response $y_s(t)$

If $f(t) = 10e^{-3t}$ we have

```
syms y(t) t tau
Dy = diff(y, 1);
ft = 10*exp(-3*t);
sys5 = diff(y,2) + 3*diff(y,1) + 2*y == 0
```

$$\text{sys5(t)} = \frac{\partial^2}{\partial t^2}\, y\,(t) + 3\,\frac{\partial}{\partial t}\, y\,(t) + 2\, y\,(t) = 0$$

```
cond1 = y(0) == 0; cond2 = Dy(0) == 1;
conds = [cond1; cond2];
y_n = dsolve(sys4, conds);
ht = diff(y_n)
```

$$\text{ht} = 2\,\mathrm{e}^{-2\,t} - \mathrm{e}^{-t}$$

Zero-State Response $y_s(t)$

```
%convolution of f(t) and h(t)
ys = int(subs(ft, tau)*subs(ht, t-tau), tau, 0, t)
```

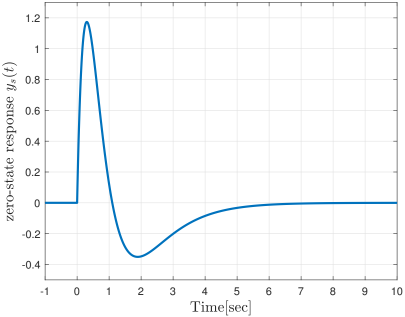
$$\mathrm{ys} = -5\,\mathrm{e}^{-3\,t}\,\left(\mathrm{e}^{2\,t} - 4\,\mathrm{e}^{t} + 3\right)$$

```
ysc = ys*heaviside(t)
```

$$\mathrm{ysc} = -5\,\mathrm{e}^{-3\,t}\,\mathrm{heaviside}\left(t\right)\,\left(\mathrm{e}^{2\,t} - 4\,\mathrm{e}^{t} + 3\right)$$

```
s = -1:0.01:10;
ysp = subs(ysc, t, ts);
plot(ts,ysp, 'linewidth', 2)
axis([-1 10 -0.5 1.3]); grid;
```

Classical Method



The last line is

$$y_s(t) = -5e^{-t} + 20e^{-2t} - 15e^{-3t},\ t \geq 0$$

Finally, the total response is $y(t) = y_0(t) + y_s(t)$.

Classical Method

Solve the differential equation

$$(D^2 + 3D + 2)y(t) = Df(t)$$

for the input $f(t) = 5t + 3$ if $y(0^+) = 2$ and $\dot{y}(0^+) = 3$.

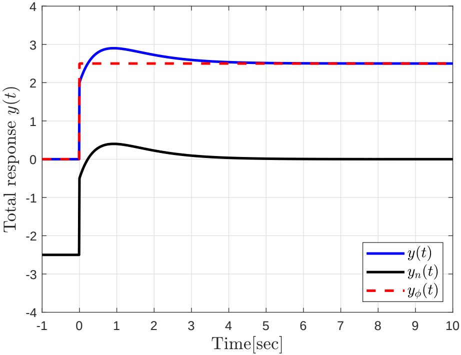
```
syms y(t) t

Dy = diff(y,1)
f = 5*t + 3;
sys1 = diff(y,2) + 3*diff(y,1) + 2*y == diff(f,t)
```

$$\text{sys1(t)} = \frac{\partial^2}{\partial t^2}\, y\,(t) + 3\,\frac{\partial}{\partial t}\, y\,(t) + 2\,y\,(t) = 5$$

Classical Method

```matlab
cond1 = y(0) == 2; cond2 = Dy(0) == 3;
conds = [cond1; cond2];
y = dsolve(sys1, conds)
```

$$y = 2\,e^{-t} - \frac{5\,e^{-2\,t}}{2} + \frac{5}{2}$$



Reference

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