INC 341 Feedback Control Systems: Lecture 6 Dynamic Response II

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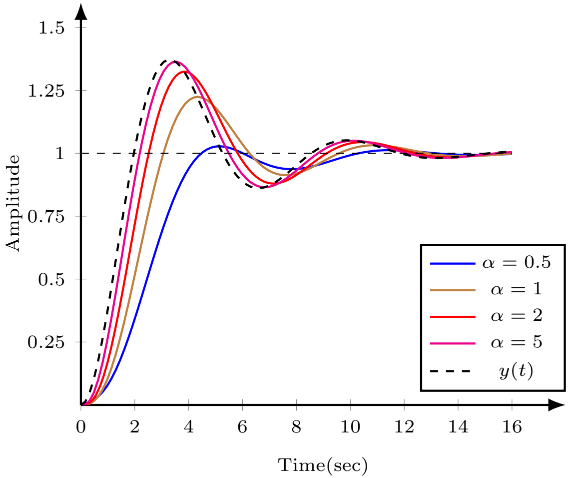
More than Two Poles, Dominant Pole Pair

- The relationship between pole locations and shape of the step response discussed in the previous section has been derived for second order systems.
- The rules could be applied to a system that has more than two poles.
- They are valid approximation when a high order system is dominated by second order dynamics.
- Consider a third order system with the transfer function

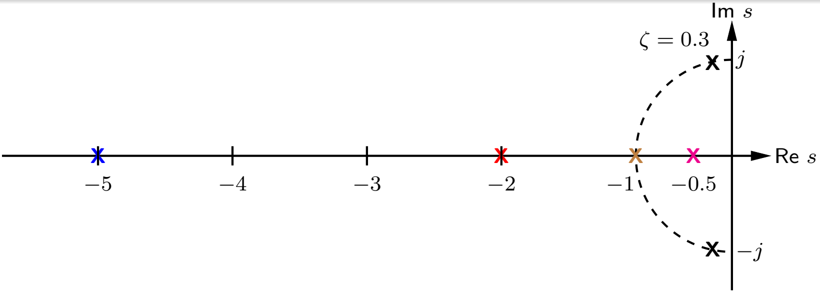
$$G(s) = \frac{K}{(s^2 + 2\zeta s + 1)\left(\frac{s}{\alpha} + 1\right)}$$

- The system can be interpreted as a standard second order system with an addition pole at $-\alpha$.
- The system responses with $\zeta = 0.3$ are shown in the next slide.

More than Two Poles, Dominant Pole Pair



More than Two Poles, Dominant Pole Pair



- The blue, brown, red, and magenta **X** are extra pole positions by using $\alpha = 0.5$, $\alpha = 1$, $\alpha = 2$ and $\alpha = 5$, respectively.
- If the extra pole, -1,-2,-5, is far to the left of the second order pole pair, it does not significantly change the behaviour of the system.
- If the extra pole (here it is -0.5) gets close to the second order pole pair, it does the response change and displays a significantly reduced overshoot.
- A system of order greater than two is said to have a *dominant pole pair* if its behaviour is dominated by a pair of complex conjugate poles and remaining poles are sufficiently far to the left.

Transfer Function Zeros
Motivation example

Consider a system

$$G_1(s) = \frac{6}{(s+2)(s+3)}.$$

The step response is $y_1(t) = \left(1 - 3e^{-2t} + 2e^{-3t}\right)\mathbb{1}(t)$. With and additional zero at $s = -1$, the transfer function becomes

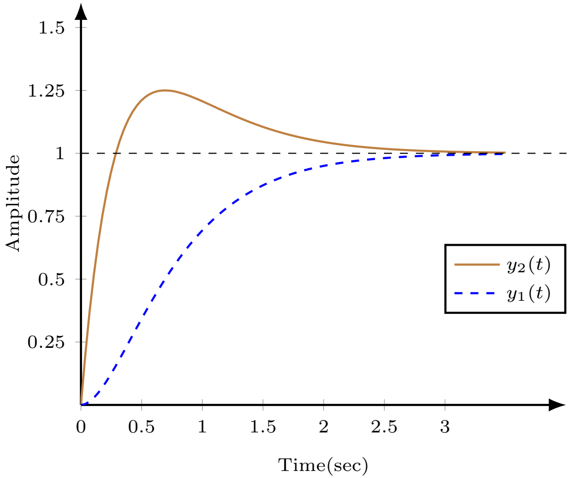
$$G_2(s) = \frac{6(s+1)}{(s+2)(s+3)}.$$

The step response is $y_2(t) = \left(1 + 3e^{-2t} - 4e^{-3t}\right)\mathbb{1}(t)$.

From the Figure (next slide), it can be seen that the presence of the zero leads to a significantly different shape.

Transfer Function Zeros

Motivation example



Transfer Function Zeros

- The poles determine the characteristic features of the transient response: whether it is fast or slow, whether or not there is oscillation in the response, and whether or not the system is stable.
- The zeros have an effect on the relative weight of each component, on how much each pole contributes to the response.
- Consider systems

$$G_1 = \frac{6}{(s+2)(s+3)}, \qquad G_2 = \frac{6(s+1)}{(s+2)(s+3)}.$$

  The main effect of adding a zero to the transfer function $G_1(s)$ is that the dominant (the slower) component e^{-2t} becomes positive and the faster component e^{-3t} negative; this results in overshoot in the step response.
- Adding a zero in the left half plane has in general a tendency to increase the overshoot.

Transfer Function Zeros

To see how can the zero on the left plane increase the overshoot, consider the two transfer functions

$$G_1(s) = \frac{b_0}{s^2 + a_1 s + a_0}, \qquad G_2(s) = \frac{b_0\left(\frac{s}{\alpha} + 1\right)}{s^2 + a_1 s + a_0}$$

- The transfer function $G_2(s)$ has the same pole and the same static gain as $G_1(s)$, but it has an additional zero at $s = -\alpha$. This zero is in the left half plane if $\alpha > 0$.
- $G_2(s)$ can also be written as

$$G_2(s) = \frac{b_0}{s^2 + a_1 s + a_0} + \frac{1}{\alpha} s \frac{b_0}{s^2 + a_1 s + a_0} = G_1(s) + \frac{1}{\alpha} s G_1(s).$$

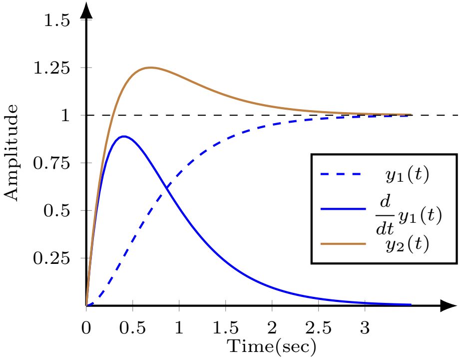
- Then the outputs are

$$Y_1(s) = G_1(s)\frac{1}{s} \qquad \text{and} \qquad Y_2(s) = G_1(s)\frac{1}{s} + \frac{1}{\alpha} s G_1(s)\frac{1}{s} = Y_1(s) + \frac{1}{\alpha} s Y_1(s).$$

In time domain, the step response of the plant with an additional zero is therefore

$$y_2(t) = y_1(t) + \frac{1}{\alpha}\dot{y}_1(t)$$

Thus, the effect of adding a left half plane zero on the response is that its derivative - scaled by $1/\alpha$ - is added to the response.

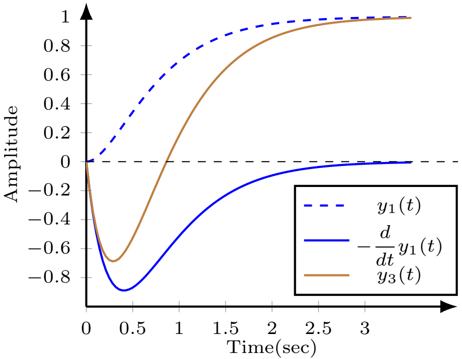


Transfer Function Zeros

Left half-plane zero

If an additional *right* half plane zero is included in the dynamics, the derivative of the step response without zero is subtracted instead of added. This causes undershoot in the step response. The system with right half plane zero is called *non-minimum phase system*.

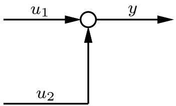
$$G_3(s) = \frac{b_0\left(-\frac{s}{\alpha}+1\right)}{s^2+a_1 s+a_0} \quad \Rightarrow \quad y_3(t) = y_1 - \frac{1}{\alpha}\dot{y}_1(t)$$



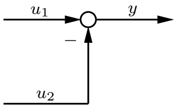
System Modelling Diagrams

Block Diagram

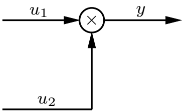
- **summing system:** $y(t) = u_1(t) + u_2(t)$



- **difference system:** $y(t) = u_1(t) - u_2(t)$



- **multiplier system:** $y(t) = u_1(t)u_2(t)$

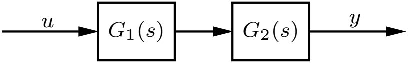


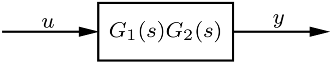
System Modelling Diagrams

Block Diagram

we can interconnect systems to form new systems, e.g.,

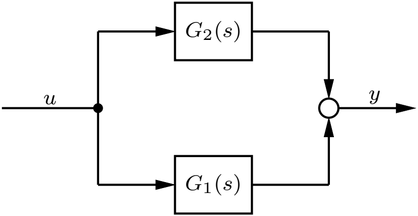
- **cascade (or series):** $y = G_2(s)(G_1(s)u) = G_2(s)G_1(s)u$

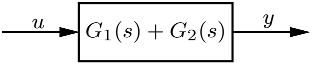




  (note the block diagrams and algebra are reversed)

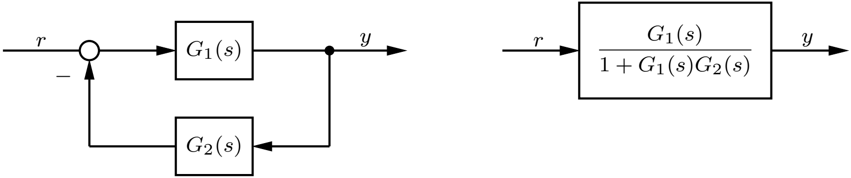
- **sum (or parralled):** $y = G_1(s)u + G_2(s)u$





- **Feedback:**

$$G_{cl}(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$



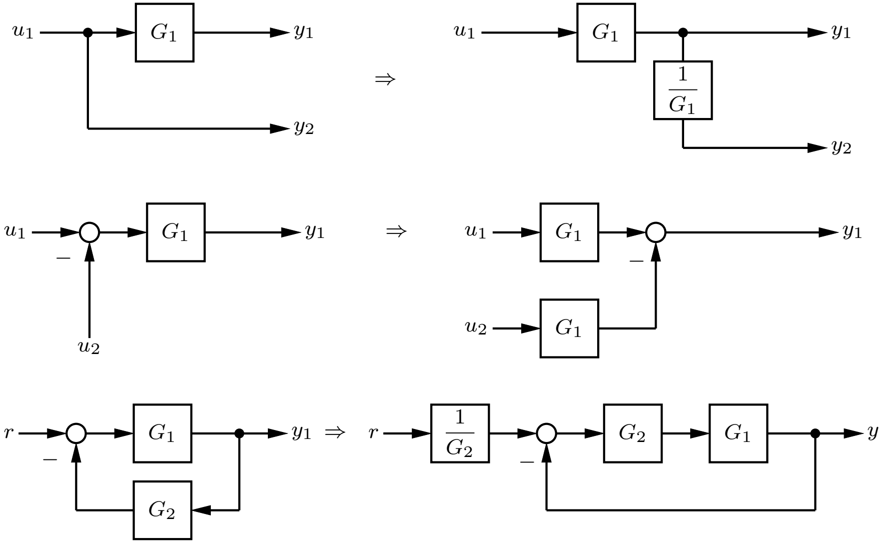
- the minus sign is for a negative feedback while the plus sign is for a positive feedback. To remember this formula is

$$\text{closed loop transfer function } = \frac{\text{forward gain}}{1 - \text{loop gain}}$$

  where *forward gain* stands for the transfer function in the forward path and *loop gain* for the transfer function seen when traversing the loop.

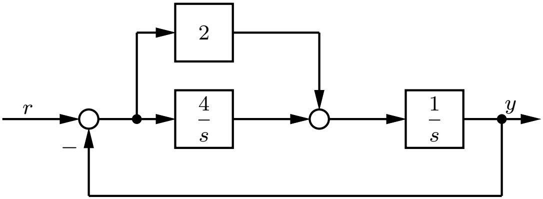
System Modelling Diagrams

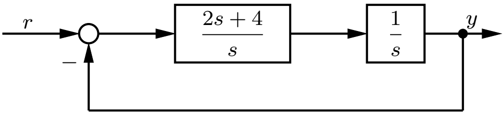
Block Diagram Algebra



Block Diagram Reduction

Simple Example





$$G_{cl} = \frac{\dfrac{2s+4}{s^2}}{1 + \dfrac{2s+4}{s^2}} = \frac{2s+4}{s^2+2s+4}$$

Transfer Function of a Simple System Using
MATLAB and SciLAB

MATLAB

```
% block diagram reduction
s = tf('s');
sysG1 = 2;
sysG2 = 4/s;

% parallel combination of G1 and G2 to form subsystem G3
sysG3 = sysG1+sysG2;

% series combination of G3 and G4
sysG4 = 1/s;
sysG5 = sysG3*sysG4;

% Unity negative feedback
[sysCL] = feedback(sysG5,1);
```

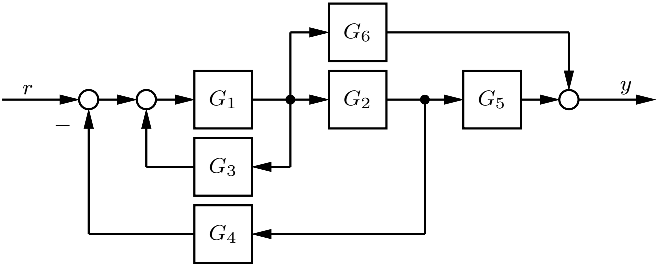
Transfer Function of a Simple System Using
MATLAB and SciLAB

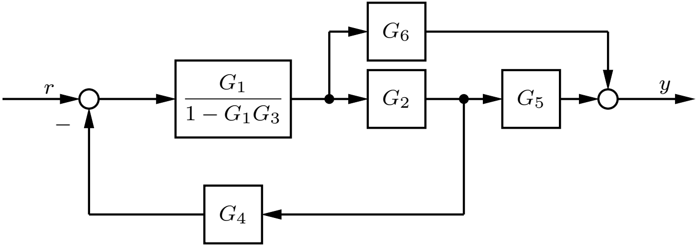
SciLAB

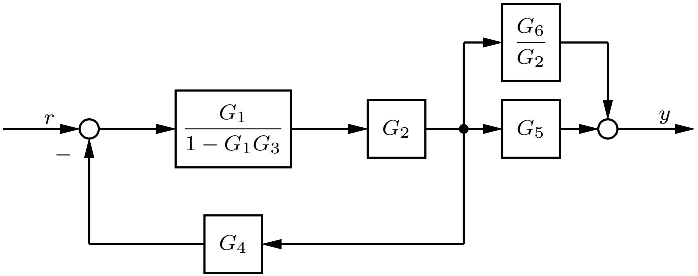
```
// block diagram reduction
s = poly([0 1],'s',"c");
sysG1 = 2;
sysG2 = 4/s;

// parallel combination of G1 and G2
sysG3 = sysG1+sysG2;

sysG4 = 1/s;
sysG5 = sysG3*sysG4;

// feedback
sysCL = sysG5/(1+sysG5)
```

Block Diagram Reduction

Simple Example







$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{G_1G_2}{1-G_1G_3}}{1+\frac{G_1G_2G_4}{1-G_1G_3}}\left(G_5+\frac{G_6}{G_2}\right)$$

$$= \frac{G_1G_2G_5+G_1G_6}{1-G_1G_3+G_1G_2G_4}$$

Reference

1. Norman S. Nise, "*Control Systems Engineering*,6^{th} edition, Wiley, 2011
2. Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini, "*Feedback Control of Dyanmic Systems*",4^{th} edition, Prentice Hall, 2002
3. Herbert Werner, "*Introduction to Control Systems*", Lecture Notes