King Mongkut's University of Technology Thonburi

Midterm examination 1st semester, academic year 2016

Subject: INC 341 Feedback Control Systems — Automation Engineering, 3rd year

Date: Thu 22 September 2016 — **Time:** 09.00 – 12.00

Warning

1. This examination contains 4 questions with 74 points, 7 pages including cover and formulas sheets;
2. Answer all questions in separated answer booklet;
3. One KMUTT approved model calculator per student is allowed;
4. The exam is closed-book.

When student finishes the examination, raise your hand to acknowledge and ask for leaving. Do not take examination paper and answer booklets out of the examination place.

Student accused of cheating at the examination will be severely punished to retire from the university.

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This paper has passed the approval from department of control system and instrumentation engineering

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Acting Head of the Department

1. Consider the transfer function

$$G(s) = \frac{0.01(s-10)}{s^2 + 1.01s + 0.01},$$

and the closed-loop system with the plant $G(s)$ and the proportional controller $C(s) = K_p$ shown in Figure 1.

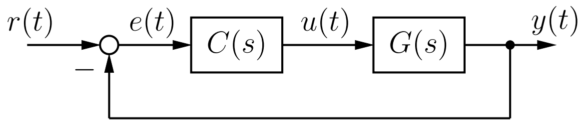


Figure 1: Closed-loop system

a) Let $K_p = -1$. Is the system stable? Give reasons by using Routh criterion. (5 points)
Solution:
If $K_p = -1$ the closed-loop transfer function is

$$G_{cl} = \frac{-0.01(s-10)}{s^2 + s + 0.11}$$

Constructing a Routh array as follow:

$$\begin{array}{c|cc} s^2 & 1 & 0.11 \\ s^1 & 1 & 0 \\ s^0 & \dfrac{-\begin{vmatrix} 1 & 0.11 \\ 1 & 0 \end{vmatrix}}{1} = 0.11 & \end{array}$$

There is no sign change, then from Routh criterion the system is stable. $\square$

b) Determine the range of $K_p \in \mathbb{R}$ for which the closed-loop system is stable. (5 points)
Solution:
The closed-loop transfer function is

$$G_{cl}(s) = \frac{K_p(0.01(s-1))}{s^2 + (1.01 + 0.01K_p)s + (0.01 - 0.1K_p)}.$$

Using Routh array

$$
\begin{array}{c|cc}
s^2 & 1 & 0.01 - 0.1K_p \\
s^1 & 1.01 + 0.01K_p & 0 \\
s^0 & \begin{array}{c} \dfrac{-\begin{vmatrix} 1 & 0.01 - 0.1K_p \\ 1.01 + 0.01K_p & 0 \end{vmatrix}}{1.01 + 0.01K_p} \\ = 0.01 - 0.1K_p \end{array} &
\end{array}
$$

To make the system stable $1.01 + 0.01K_p$ and $0.01 - 0.1K_p$ must be positive. Then

$$
\begin{aligned}
&0.01 - 0.1K_p > 0 \text{ and } 1.01 + 0.01K_p > 0 \\
&-101 < K_p < 0.1
\end{aligned}
$$

□

c) In Figure 2 the step responses of the closed-loop system with different proportional controllers are shown. Assign the following controllers

(i)	$K_p = -95$	(ii)	$K_p = -20\pi$
(iii)	$K_p = 2.5$	(iv)	$K_p = -5$

to the matching figures and give reasons for your choice. (10 points)

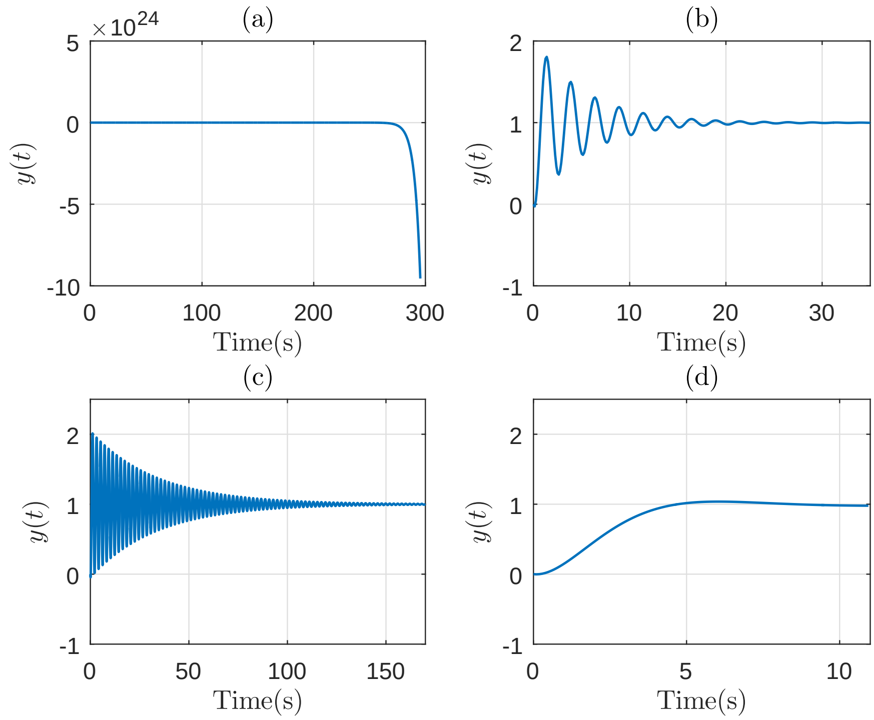


Figure 2: Step response

Solution:
From (b) K_p must be less than 0.1, it is obvious that $K_p = 2.5$ leads the closed-loop be unstable and (iii) is matching with Figure (a).

For negative values of K_p, we have the following analysis:

(i) $K_p = -9.5$ The characteristic equation of the closed-loop system is

$$s^2 + (0.06)s + 9.51 = 0$$
$$\omega_n = \sqrt{9.51} = 3.08$$
$$\zeta = \frac{0.06}{2(3.08)} = 0.0098$$

(ii) $K_p = -20\pi$ The characteristic equation of the closed-loop system is

$$s^2 + 0.382s + 6.27 = 0$$
$$\omega_n = \sqrt{6.27} = 2.5$$
$$\zeta = \frac{0.382}{2(2.5)} = 0.0764$$

(iv) $K_p = -5$ The characteristic equation of the closed-loop system is

$$s^2 + 0.96s + 0.51 = 0$$
$$\omega_n = \sqrt{0.51} = 0.714$$
$$\zeta = \frac{0.96}{2(0.714)} = 0.672$$

From the damping ratios and the natural frequencies, we have $(i) = (c)$, $(ii) = (b)$, and $(iv) = (d)$. □

2. Consider the system with transfer function

$$G(s) = \frac{1}{(s+1)(0.5s+1)}.$$

The system $G(s)$ is operated as plant in a control loop with a controller $C(s)$ shown in Figure 3.

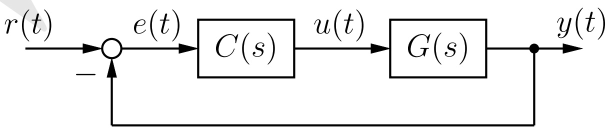


Figure 3: Closed-loop system

a) Is it possible to reach the desired damping ratio $\zeta > 0.7$ using the given controller $C(s) = K_p > 0$? What is the maximum value of K_p? (6 points)

Solution:
For $K_p > 0$, the closed-loop transfer function is

$$G_{cl}(s) = \frac{K_p}{0.5s^2 + 1.5s + (1 + K_p)},$$

The characteristic equation is

$$s^2 + 3s + 2(1 + K_p) = 0, \qquad \omega_n = \sqrt{2 + 2K_p}$$
$$\zeta = \frac{3}{2\sqrt{2 + K_p}}$$

We need $\zeta > 0.7$ then

$$3 = 1.4\sqrt{2 + 2K_p}$$
$$K_p = 1.295$$

To get $\zeta > 0.7$, we need $K_p < 1.295$. $\square$

b) If we want to have the peak overshoot $M_p = 16.7\%$, what is the maximum value of K_P for $M_p < 16.7\%$? What is the steady-state error $e_{ss}(\infty)$ of the closed-loop system responding to a unit step input $\mathbb{1}(t)$ (8 points)
Solution:
Since $M_p \approx 1 - \frac{\zeta}{0.6}$, we have $\zeta = (0.933)(0.6) = 0.56$. From part (a)

$$\omega_n = \sqrt{2 + 2K_p}$$
$$\zeta = \frac{3}{2\sqrt{2 + 2K_p}}$$
$$K_p = 3.5$$

The closed-loop transfer function is

$$\frac{E(s)}{R(s)} = \frac{0.5s^2 + 1.5s + 1}{0.5s^2 + 1.5s + 4.5}$$
$$= \frac{s^2 + 3s + 2}{s^2 + 3s + 9}$$
$$e_{ss}(\infty) = \lim_{s \to 0} s \frac{E(s)}{R(s)} \frac{1}{s} = \frac{2}{9} = 0.22$$

$\square$

c) We can reduce the steady-state error without increasing overshoot by using PD controller:

$$C(s) = K_P + T_D s.$$

With a unit step input, find the values of K_p and T_D for the closed-loop system shown in Figure 3 to have the steady-state error $e_{ss}(\infty)$ less than 1% while the maximum overshoot M_p is less 16.7%. (10 points)

Solution:

We need $e_{ss}(\infty) < 1\% = 0.01$. From

$$L(s) = \frac{K_p + T_d s}{(s+1)(0.5s+1)}$$

The steady-state error is

$$\frac{1}{1+K_p} < 0.01,$$

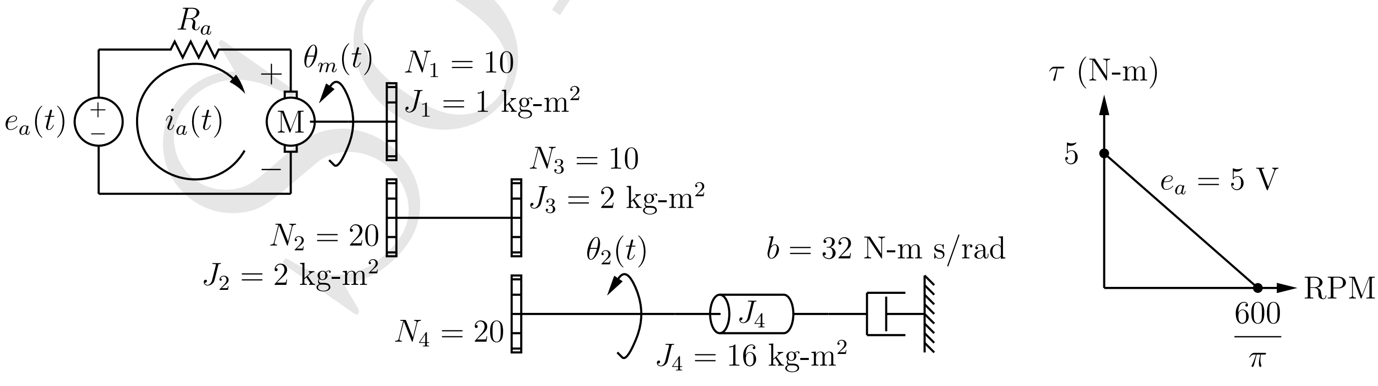
Then $K_p = \dfrac{0.99}{0.01} = 99$

To get $M_p < 16.7\%$ or $\zeta > 0.56$ (from part (b))

$$\begin{aligned}
G_{cl}(s) &= \frac{L(s)}{1+L(s)} = \frac{K_p + T_D s}{(s+1)(0.5s+1) + K_p + T_D s} \\
&= \frac{2(K_p + T_D s)}{s^2 + (3+2T_D)s + (2+2K_p)} \\
\omega_n &= \sqrt{2+2(99)} = 14.14 \\
\zeta &= \frac{3+2T_d}{2(14.14)} = 0.56 \\
T_D &= 6.42.
\end{aligned}$$

□

3. The motor whose torque-speed characteristics are shown in Figure 4 drives the load shown in the diagram. Some of the gears have inertia.



(a) **(b)**

Figure 4: Motor system

a) Find the transfer functions $\Theta_m(s)/E_a(s)$ and $\Theta_2(s)/E_a(s)$. (6 points)

Solution: From the torque-curve characteristic, we have

$$\frac{K_t}{R_a} = \frac{\tau_{\text{stall}}}{e_a} = 1, \qquad K_b = \frac{e_a}{\omega_{\text{no-load}}} = \frac{1}{4}$$

$$\begin{aligned} J_m &= J_1 + (J_3 + J_2)\left(\frac{N_1}{N_2}\right)^2 + J_4\left(\frac{N_1}{N_2}\right)^2\left(\frac{N_3}{N_4}\right)^2 = 3 \\ b_m &= b\left(\frac{N_1}{N_2}\right)^2\left(\frac{N_3}{N_4}\right)^2 = 2 \end{aligned}$$

We have

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{K_t(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(b_m + \frac{K_t K_b}{R_a}\right)\right]} = \frac{1/3}{s(s+0.75)}$$

Since $\theta_2 = 1/4\theta_m$, then

$$\frac{\Theta_2(s)}{E_a(s)} = \frac{1/12}{s(s+0.75)}$$

$\square$

b) From the results of part (a), find the transfer functions that relates the speed of motor $\dot{\theta}_m(t)$ to the input armature voltage $e_a(t)$. (2 points)
Solution:
Since $\mathcal{L}\{\dot{\theta}_m(t)\} = s\Theta_m(s)$, hence

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{1/3}{s+0.7}$$

$\square$

c) To control the angle $\theta_2(t)$ using the transfer function from part (a) in a closed-loop configuration, which controller between

$$C_1(s) = K_p(1 + T_d s) \quad \text{and} \qquad C_2(s) = K_p\left(1 + \frac{1}{T_I s}\right)$$

is the best selection to use regarding the steady-state error and transient response? Explain your answer.(5 points)
Solution:
The closed-loop system $\Theta_m(s)/E_a(s)$ is a Type I system, then for the step input (position control) the $e_{ss}(\infty)$ is always zero. To improve the transient response, we can use D term in $C_1(s)$, while $C_2(s)$ cannot improve the damping ratio. In this case the best controller is $C_1(s)$. $\square$

4. Assume all operational amplifiers in the circuit of Figure 5 are ideal.

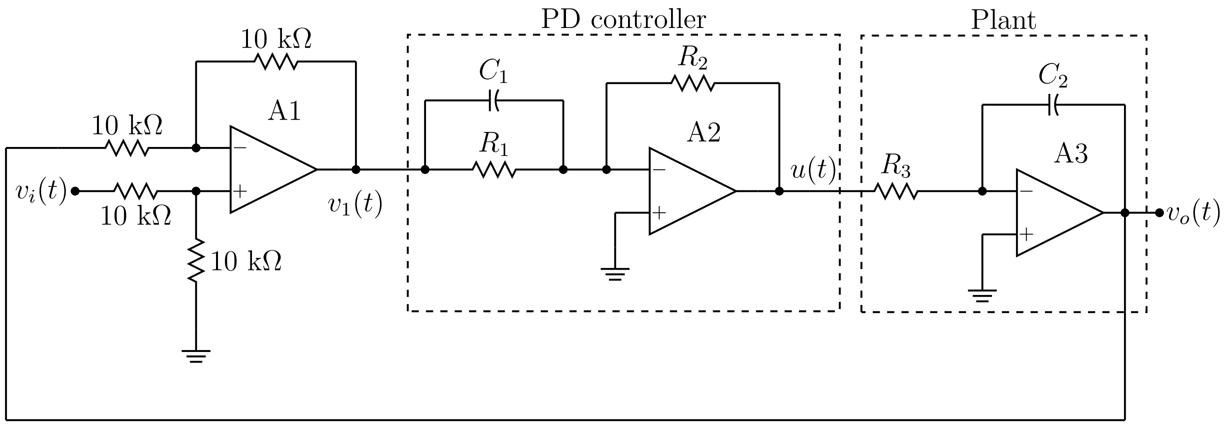


Figure 5: Op-Amp circuit

a) Show that the operation amplifier A1 is a subtracting amplifier. Namely, $v_1(t) = v_i(t) - v_o(t)$. (3 points)

Solution:

At A1, let $v_x(t)$ is the virtual short voltage at the two input leg of the Op-amp. We obtain

$$v_x(t) = \frac{v_i(t)}{2}, \qquad \frac{v_x(t) - v_1(t)}{R} = \frac{v_o(t) - v_x(t)}{R}$$
$$v_x(t) = \frac{v_o(t) + v_1(t)}{2}$$

Then we have $v_i(t) = v_o(t) + v_i(t)$ or $v_1(t) = v_i(t) - v_o(t)$. □

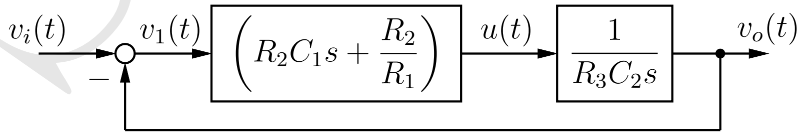
b) Find the transfer function $U(s)/V_1(s)$ and $V_o(s)/U(s)$. (4 points)

Solution:

$$\frac{U(s)}{V_1(s)} = -\left(R_2C_1s + \frac{R_2}{R_1}\right)$$
$$\frac{V_o(s)}{U(s)} = -\frac{1}{R_3C_2}$$

c) Draw a block diagram of the whole system, with the subtracting amplifier (A1) as a summing junction. (3 points)

Solution:



□

d) Find the closed-loop transfer function $V_o(s)/V_i(s)$. (3 points)

Solution:

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 C_1 s + \dfrac{R_2}{R_1}}{\left(R_3 C_2 + R_2 C_1\right) s + \dfrac{R_2}{R_1}}$$
$$= \frac{R_1 R_2 C_1 s + R_2}{\left(R_1 R_3 C_2 + R_1 R_2 C_1\right) s + R_2}$$

□

e) What is the steady-state gain of the closed-loop system? (2 points)
Solution: From part (d), we have

$$G_{cl}(0) = 1$$

□

f) What is the time constant τ of the closed-loop system from part (d) in terms of R_i and C_i, where $i = 1, 2, 3$? (2 points)
Solution:
From the closed-loop transfer function

$$\tau = \frac{R_1 R_3 C_2 + R_1 R_2 C_1}{R_2}$$

□

Formulas and Tables

Laplace Transform

$f(t)$	$F(s)$
$f(t)$	$\int_0^\infty f(t)e^{-st}dt$
$\delta(t)$	1
$\mathbb{1}(t)$	$\frac{1}{s}$
$t^n\mathbb{1}(t)$	$\frac{n!}{s^{n+1}}$
$e^{\lambda t}\mathbb{1}(t)$	$\frac{1}{s-\lambda}$
$\frac{df}{dt}$	$sF(s) - f(0^-)$
$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - \dot{f}(0^-)$

Final value theorem: $f(\infty) = \lim_{s\to 0} sF(s)$

DC Motor

- **Torque-speed Curve:**

$$\frac{R_a}{K_t}\tau_m(t) + K_b\omega_m(t) = e_a(t)$$

$$\frac{K_t}{R_a} = \frac{\tau_{\text{stall}}}{e_a}, \qquad K_b = \frac{e_a}{\omega_{\text{no-load}}}$$

- **Transfer function:**

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_aJ_m)}{s\left[s + \frac{1}{J_m}\left(b_m + \frac{K_tK_b}{R_a}\right)\right]}$$

Gear

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{\tau_1}{\tau_2}$$

Second-order System:

General form: $G(s) = K\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Rise time: $t_r = \dfrac{1.8}{\omega_n}$, Maximum overshoot: $\%M_p = \left(1 - \dfrac{\zeta}{0.6}\right) \times 100$

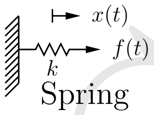
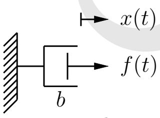
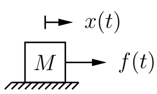
Settling time: $t_s = \dfrac{4.6}{\zeta\omega_n}$

Electrical Network

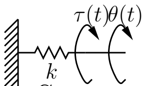
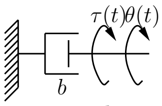
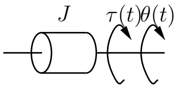
Component	Voltage-current	Current-voltage	Impedance $Z(s) = V(s)/I(s)$
Capacitor	$v(t) = \dfrac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\dfrac{d}{dt}v(t)$	$\dfrac{1}{Cs}$
Resistor	$v(t) = Ri(t)$	$i(t) = \dfrac{1}{R}v(t)$	R
Inductor	$v(t) = L\dfrac{d}{dt}i(t)$	$i(t) = \dfrac{1}{L}\int_0^t v(\tau)d\tau$	Ls

Note: All initial conditions are zero.

Translational Mechanical System

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
 Spring	$f(t) = k\int_0^t v(\tau)d\tau$	$f(t) = kx(t)$	k
 Viscous damper	$f(t) = bv(t)$	$f(t) = b\dot{x}(t)$	bs
 Mass	$f(t) = M\dot{v}(t)$	$f(t) = M\ddot{x}$	Ms^2

Rotational Mechanical System

Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = \hat{\tau}(s)/\Theta(s)$
 Spring	$\tau(t) = k \int_0^{t_1} \omega(t)dt$	$\tau(t) = k\theta(t)$	k
 Viscous damper	$\tau(t) = b\omega(t)$	$\tau(t) = b\dot{\theta}(t)$	bs
 Inertia	$\tau(t) = J\dot{\omega}(t)$	$\tau(t) = J\ddot{\theta}$	Js^2