INC 341 Feedback Control Systems **Home Work 3**
Assign: 9 Sep 2016 **Due for 3AE, 3A, 3WIL**: 15 Sep 2016

Instruction: Hand in your work with name and code by hand before the class is started. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. For the electric circuit shown in Fig. 1, **find** the following:

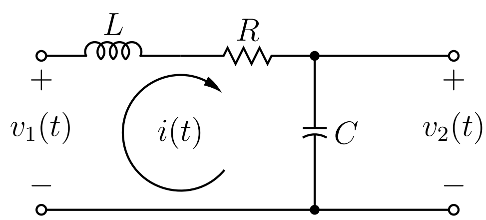


Figure 1: Problem 1.

a) The time-domain equation relating $i(t)$ and $v_1(t)$. (2 points)
Solution:
We have

$$-v_1(t) + L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = 0$$
$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i(t) = \frac{1}{L}\frac{dv_1}{dt}$$

□

b) The time-domain equation relating $i(t)$ and $v_2(t)$. (2 points)
Solution:
We have

$$i_C(t) = i(t) = C\frac{dv_2}{dt}$$
$$\frac{dv_2}{dt} = \frac{1}{C}i(t)$$

□

c) Assuming all initial conditions are zero, the transfer function $V_2(s)/V_1(s)$ and the damping ratio ζ and undamped natural frequency ω_n of the system. (3 points)

Solution:
The Laplace transform of the both above equations are

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) I(s) = \frac{1}{L}sV_1(s)$$
$$sV_2(s) = \frac{1}{C}I(s).$$

Then

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) CsV_2(s) = \frac{1}{L}sV_1(s)$$
$$\frac{V_2(s)}{V_1(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

□

d) The values of R that will result in $v_2(t)$ having an overshoot of no more than 25%, assuming $v_1(t)$ is a unit step, $L = 10$ mH, and $C = 4\mu$ F. (3 points)
Solution: Since $M_p = 0.25$, and

$$0.25 = 1 - \frac{\zeta}{0.6} \qquad \Rightarrow \qquad \zeta = 0.45.$$

From $V_2(s)/V_1(s)$ of the previous part

$$\frac{R}{L} = 2\zeta\omega_n$$
$$\omega_n = \sqrt{1/LC} = 5 \times 10^3$$
$$\zeta = (R/L)/(2(5 \times 10^3)) = \frac{R}{100} = 0.45$$
$$R = 45\ \Omega$$

□

2. The equations of motion for the DC motor is given as

$$J_m\ddot{\theta}_m + \left(b + \frac{K_tK_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a}v_a,$$

where $J_m = 0.01$ kg·m^2, $b = 0.001$ N·m·sec, $K_e = 0.02$ V·sec, $K_t = 0.02$ N·m/A, $R_a = 10\ \Omega$.

(a) Find the transfer function between the applied voltage v_a and the motor speed $\dot{\theta}_m$. (2 points)

Solution: Substituting all values and taking Laplace transform, we have

$$0.01s^2\Theta_m(s) + \left(0.001 + \frac{0.02(0.02)}{10}\right) s\Theta_m(s) = \frac{0.02}{10}V_a(s)$$

$$\left(s^2 + \left(0.1 + \frac{0.02}{5}\right) s\right) \Theta_m(s) = 0.2V_a(s)$$

$$\frac{s\Theta_m(s)}{V_a(s)} = \frac{\Omega(s)}{V_a(s)} = \frac{0.2}{s + 1.04}$$

□

(b) What is the steady-state speed of the motor after a voltage $v_a = 10$ V has been applied? (1 point)
Solution

$$G_{cl}(0) = 0.2/1.04 = 0.192$$

The steady-state speed of the motor after a voltage $v_a = 10$ V is $10(0.192) = 1.92$ rad/sec. □

(c) Find the transfer function between the applied voltage v_a and the shaft angle θ_m (2 points)
Solution: From the previous solution, we have

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{0.2}{s(s + 1.04)}$$

□

(d) Suppose feedback is added to the system in part (c) so that it becomes a position servo device such that the applied voltage is given by

$$v_a = K(\theta_r - \theta_m),$$

where K is the feedback gain. Find the transfer function between θ_r and θ_m. (3 points) (**Hint:** Draw your block diagram of the closed-loop system first.)
Solution:
The closed-loop transfer function is

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{0.2K}{s^2 + 1.04s + 0.2K}$$

□

(e) What is the maximum value of K that can be used if an overshoot $M_p < 20\%$ is desired? (3 points)

Solution:
From

$$M_p = 1 - \frac{\zeta}{0.6},$$

we have $\zeta = -(0.2 - 1)0.6 = 0.48$. We have

$$2\zeta\omega_n = 1.04$$
$$\omega_n = \frac{1.04}{2(0.48)} = 1.08$$

Since $\omega_n^2 = 0.2K$, we have

$$0.2K = 1.08^2 = 1.17$$
$$K = 5.85$$

□

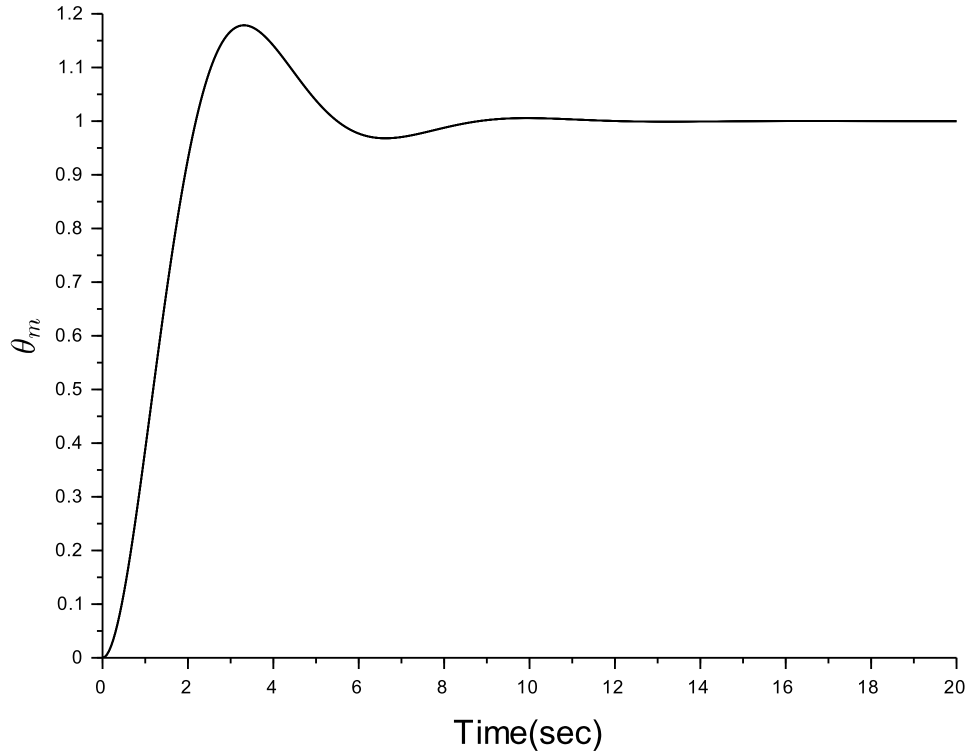
(f) What values of K will provide a rise time of less than 4 sec? (Ignore the M_p constraint.) (2 points)
Solution: From

$$t_r = \frac{1.7}{\omega_n},$$

then we need $\omega_n > 1.7/4 = 0.425$ rad/sec. The gain K must be greater than $0.425^2/0.2 = 0.9$. □

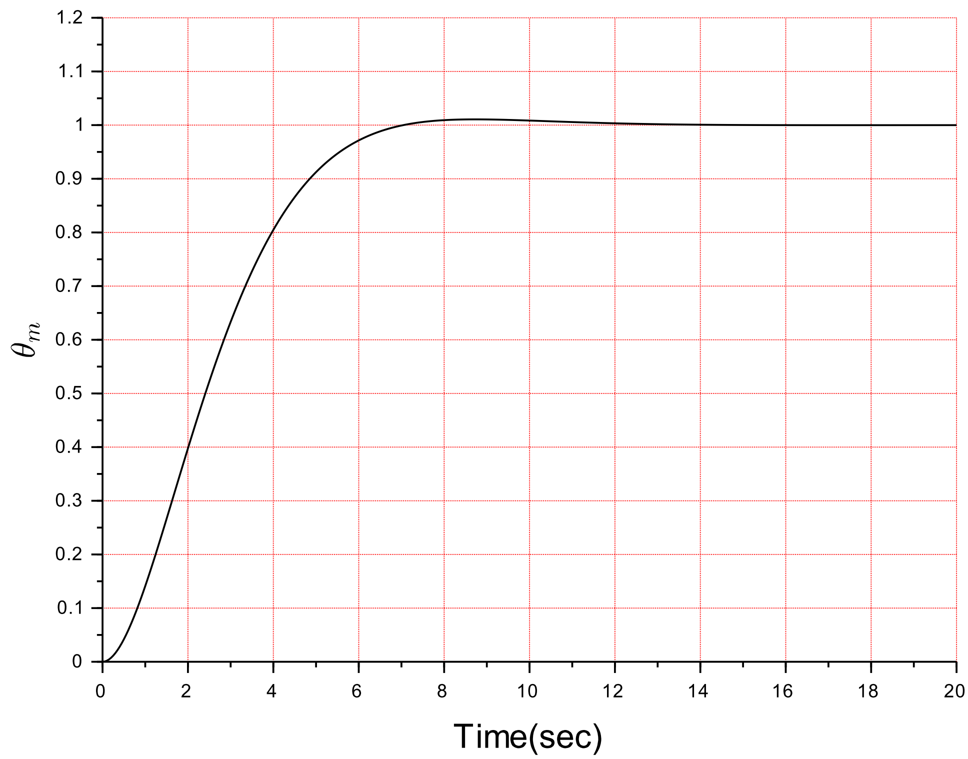
(g) Prove your designs in parts (e) and (f) by using SciLab. (Show your codes and plots) (3 points)
Solution:
Part (e):

```
  // hw 3 part e
s = poly(0,'s');
K = 5.85;
Gcl = 0.2*K/(s^2 + 1.04*s + 0.2*K);
t = 0:0.05:20;
y = csim('step',t,Gcl)
plot2d(t,y)
xlabel('Time(sec)','fontsize',4);
ylabel('$\theta_m$','fontsize',4)
```



From simulation we need $K > 2$ at least.

```
Part f:// hw 3 part f
s = poly(0,'s');
K = 2;
Gcl = 0.2*K/(s^2 + 1.04*s + 0.2*K);
t = 0:0.05:20;
y = csim('step',t,Gcl)
plot2d(t,y)
xlabel('Time(sec)','fontsize',4);
ylabel('$\theta_m$','fontsize',4)
xgrid(5,0.5,7)
```



1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_m
0
2
4
6
8
10
12
14
16
18
20
Time(sec)