INC 341 Feedback Control Systems **Home Work 2**
Assign: 27 Aug 2016 **Due for 3AE, 3A, 3WIL**: 5 Sep 2016

Instruction: Hand in your work with name and code by hand before the class is started. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. In the system shown in Figure 1, the inertia, J, of radius, r, is constrained to move only about the stationary axis A. A viscous damping force of the translational value b exists between the bodies J and M. If an external force, $f(t)$, is applied to the mass, find the transfer function, $G(s) = \Theta(s)/F(s)$.

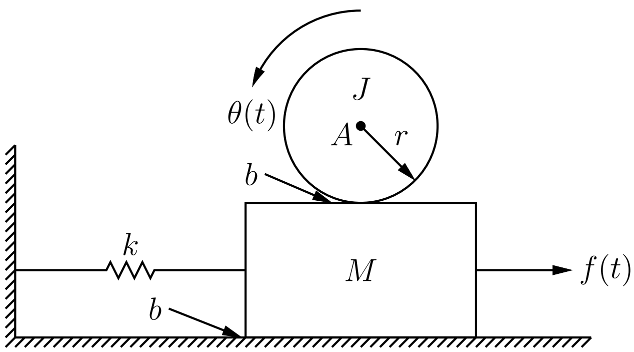
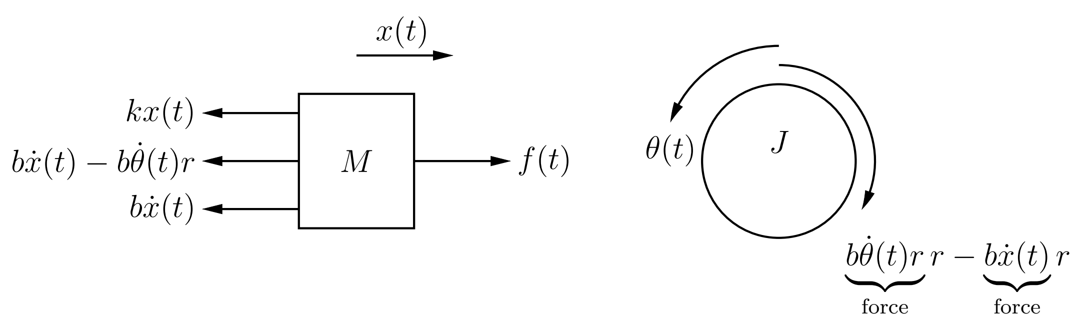


Figure 1: Problem 1.

Solution: Draw free body diagrams



The dynamic equations are ($\sum F = ma$ and $\sum \tau = J\alpha$)

$$\left(Ms^2 + 2bs + k\right) X(s) - brs\Theta(s) = F(s)$$
$$-brsX(s) + \left(Js^2 + br^2 s\right) \Theta(s) = 0.$$

In matrix form, we have

$$\begin{bmatrix} Ms^2 + 2bs + k & -brs \\ -brs & Js^2 + br^2 s \end{bmatrix} \begin{bmatrix} X(s) \\ \Theta(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}$$

Then,

$$\begin{aligned}
\begin{bmatrix} X(s) \\ \Theta(s) \end{bmatrix} &= \frac{1}{(Ms^2 + 2bs + k)(Js^2 + br^2 s) - b^2 r^2 s^2} \begin{bmatrix} Js^2 + br^2 s & brs \\ brs & Ms^2 + bs + k \end{bmatrix} \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \\
\frac{\Theta(s)}{F(s)} &= \frac{brs}{JMs^4 + \left(2Jb + Mbr^2\right) s^3 + \left(Jk + b^2 r^2\right) s^2 + kbr^2 s} \\
&= \frac{br}{JMs^3 + \left(2Jb + Mbr^2\right) s^2 + \left(Jk + b^2 r^2\right) s + kbr^2}
\end{aligned}$$

$\square$

2. The motor whose torque-speed characteristics are shown in Figure 2 drives the load shown in the diagram. Some of the gears have inertia. Find the transfer function, $G(s) = \Theta_2(s)/E_a(s)$. (10 points)

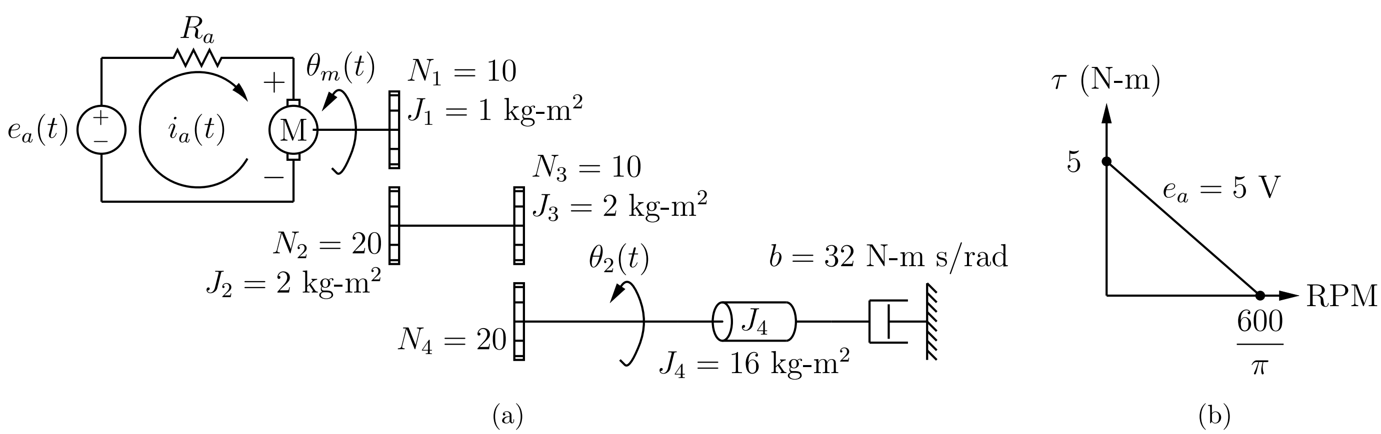


Figure 2: Problem 2.

Solution:

From the torque-speed curve, we have

$$\frac{K_t}{R_a} = \frac{\tau_{\text{stall}}}{e_a} = \frac{5}{5} = 1 \qquad \text{and} \qquad K_b = \frac{e_a}{\omega_{\text{no-load}}} = \frac{5}{\frac{600}{\pi} 2\pi \frac{1}{60}} = \frac{5}{20} = \frac{1}{4}$$

From the motor, we have

$$\begin{aligned}
J_m &= J_1 + (J_2 + J_3)\left(\frac{N_1}{N_2}\right)^2 + J_4\left(\frac{N_1}{N_2}\right)^2\left(\frac{N_3}{N_4}\right)^2 = 1 + 4\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = 3 \\
b_m &= b\left(\frac{N_1}{N_2}\right)^2\left(\frac{N_3}{N_4}\right)^2 = 32\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = 2,
\end{aligned}$$

Thus,

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(b_m + \frac{K_t K_b}{R_a}\right)\right]} = \frac{1/3}{s\left(s + \dfrac{1}{3}\left(2 + \dfrac{1}{4}\right)\right)} = \frac{1/3}{s(s+0.75)}$$

Since $\theta_2(t) = \dfrac{1}{4}\theta_m(t)$, then

$$\frac{\Theta_2(s)}{E_a(s)} = \frac{1/12}{s(s+0.75)}$$

□