INC 341 Feedback Control Systems **Home Work 1**

Assign: 10 Aug 2016 **Due for 3AE, 3A, 3WIL**: 23 Aug 2016 (2 weeks)

Instruction: Hand in your work with name and code by hand before the class is started. DO NOT copy homework from your classmates or lend it to others. Anyone who violates this regulation will be given -10 for the homework.

1. Compute the transfer function from $u(t)$ to $i(t)$ of the network shown in Figure 1. (10 points)

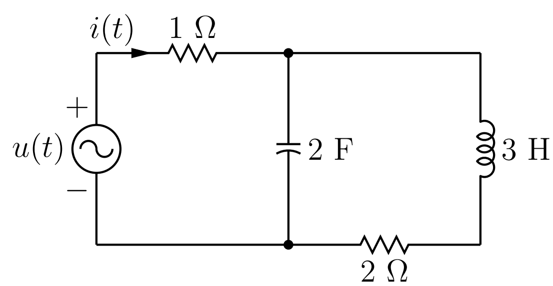
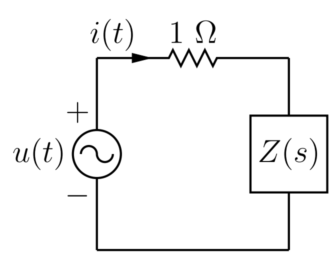


Figure 1: Network.

Solution:

Using an impedance method, we change the circuit as shown in Figure below: Hence,



$$Z(s) = \frac{1}{2s}||(3s+2) = \frac{3s+2}{6s^2+4s+1}$$

Using this $Z(s)$, we have

$$U(s) = \left(1 + \frac{3s+2}{6s^2+4s+1}\right) I(s)$$

$$\frac{I(s)}{U(s)} = \frac{6s^2+4s+1}{6s^2+7s+3}$$

□

2. Obtain the transfer function $E_o(s)/E_i(s)$ of the op-amp circuit shown in Figure 2. **Note:** $E_o(s)$ and $E_i(s)$ are the Laplace Transform of $e_o(t)$ and $e_i(t)$ respectively. (10 points)

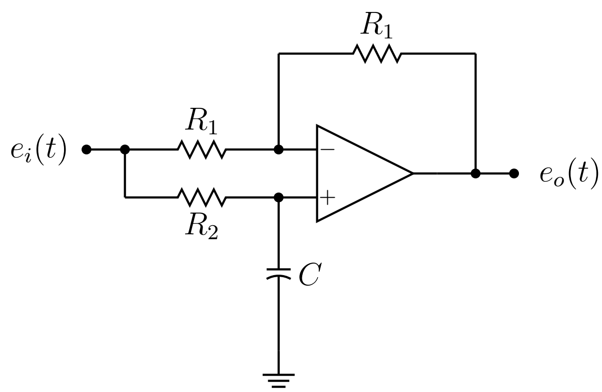


Figure 2: Operational amplifier circuit

Solution: Denote voltages at the negative and positive pin of the op-amp as E_A and E_B respectively. Using an idea of virtual short, we have

$$E_B(s) = \left(R_2 + \frac{1}{Cs}\right) I_+(s), \text{ where } i_+ \text{ is current through } R_2 \text{ and } C.$$

$$I_+(s) = \frac{E_B(s)}{1/Cs}$$

$$E_B(s) = \frac{1}{R_2Cs + 1}.$$

The upper part of the op-amp, we have

$$-e_i(t) + 2R_1 i_-(t) + e_o(t) = 0, \text{ where } i_- \text{ is current through } R_1.$$

$$i_-(t) = \frac{1}{2}\frac{e_i(t) - e_o(t)}{R_1}$$

$$E_A(t) = e_o(t) + 2R_1 i_-(t) = e_o(t) + \frac{1}{2}(e_i(t) - e_o(t))$$

$$E_A(s) = -\frac{1}{2}\left(E_o(s) - E_i(s)\right)$$

Since $E_A(s) = E_B(s)$, then

$$\left(\frac{1}{R_2Cs + 1} - \frac{1}{2}\right) E_i(s) = \frac{1}{2}E_o(s)$$

$$\frac{E_o(s)}{E_i(s)} = -\frac{R_2Cs - 1}{R_2Cs + 1} = -\frac{s - \frac{1}{R_2C}}{s + \frac{1}{R_2C}}$$

□