INC 212 Signals and System **Homework 1**
Assign: 21 April 2020 **Due Fri 24 April 2020**

Instruction: This is an in class assignment.
Member:

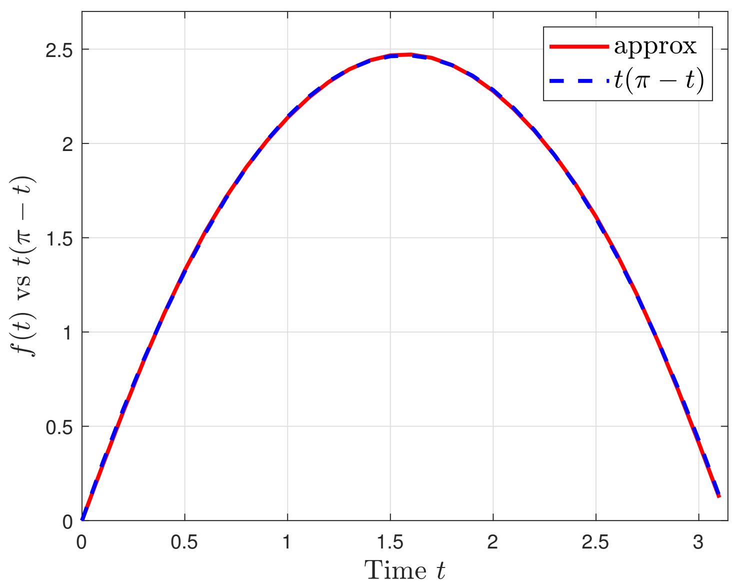
1. Name:________________________________Code:______________

Questions: Fourier Series

1. Plot along with Matlab code the first three partial sums and the function $f(t) = t(\pi - t)$:

$$f(t) = t(\pi - t) = \frac{8}{\pi}\left(\frac{\sin(t)}{1} + \frac{\sin(3t)}{27} + \frac{\sin(5t)}{125}\right), \qquad 0 < t < \pi, \tag{1}$$

Why is $1/k^3$ the decay rate for this function? What is its second derivative?
Solution: The plot and Matlab code is shown below:



Matlab code

```
t = 0:0.1:pi;
ft = (8/pi)*(sin(t)/1 + sin(3*t)/27 + sin(5*t)/125);
y1 = t.*(pi -t);
plot(t,ft,'r-', t, y1,'b--', 'linewidth',2)
```

Since $f(t)$ is an odd function then the fourier approximate contains only odd terms. Moreover

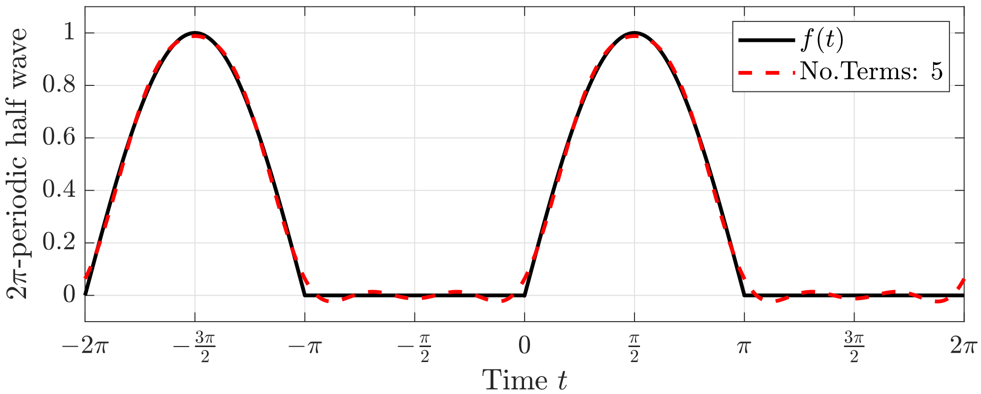
$$[1 \quad 27 \quad 125] = [1 \quad 3^3 \quad 5^3] = k^3, k = 1, 3, 5, \ldots.$$

So the decay rate is $1/k^3$. Its second derivative is

$$\frac{d^2 f(t)}{dt^2} = -\frac{8}{\pi}\left(\frac{\sin(t)}{1} + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5}\right)$$

$\square$

2. Sketch **the 2π-periodic half wave** with $f(t) = \sin(t)$ for $0 < t < \pi$ and $f(t) = 0$ for $-\pi < t < 0$. Find its Fourier series by hand and Matab by showing at least 5 first terms.
Solution:



Calculating by hand, we start with three formulas:

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)dt, \quad a_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cos(k\omega_0 t)dt, \quad b_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\sin(k\omega_0 t)dt$$

Since $\omega_0 = 2\pi/T_0$ and $T_0 = 2\pi$, then

$$\begin{aligned}
a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sin(t)dt = \frac{1}{2\pi}\int_{0}^{\pi}\sin(t)dt = \frac{1}{2\pi}\left(-\cos(t)\Big|_0^{\pi}\right) = \frac{1}{\pi}\\
a_k &= \frac{2}{2\pi}\int_{-\pi}^{\pi}\sin(t)\cos(kt)dt = \frac{1}{\pi}\int_{0}^{\pi}\sin(t)\cos(kt)dt\\
&= \frac{1}{2\pi}\int_{0}^{\pi}\left[\sin((k+1)t) + \sin((1-k)t)\right]dt\\
&= \begin{cases} 0, & k = \text{ odd number}\\ \frac{-2}{\pi(1-k^2)}, & k = \text{even number}\end{cases}\\
b_k &= \frac{2}{2\pi}\int_{-\pi}^{\pi}\sin(t)\sin(kt)dt = \frac{1}{\pi}\int_{0}^{\pi}\sin(t)\sin(kt)dt\\
&= \frac{1}{\pi}\int_{0}^{\pi}\cos((1-k)t) - \cos((1+k)t)dt\\
&= \begin{cases} \frac{1}{2}, & k = 1\\ 0, & k > 1\end{cases}.
\end{aligned}$$

The 5 first terms are shown in the Table 1.

Table 1: Five first terms of Fourier coefficient

No.	Matlab		Hand	
	a_k	b_k	a_k	b_k
0	0.3183	0	$\frac{1}{\pi} = 0.3183$	0
1	0	0.5	0	0.5
2	-0.2122	0	$\frac{-2}{3\pi} = -0.2122$	0
3	0	0	0	0
4	-0.0424	0	$\frac{-2}{15\pi} = -0.0424$	0
5	0	0	0	0

Matlab code

```
close all; clear all;
dx = 0.001; L = 2*pi; t = -L:dx:L;
n = length(t); nquart = floor(n/4); % one fourth of data point;

f = zeros(size(t));
f(1:nquart) = sin(t(1:nquart));
f(nquart+1:2*nquart) = 0;
f(2*nquart+1:3*nquart) = sin(t(2*nquart+1:3*nquart));
f(3*nquart+1:4*nquart) = 0;

for N = 1:5,
    hold off;
    plot(t, f, 'k', 'linewidth', 2)
    grid;
    % calculate only half then 2/L -> 1/L
    A0 = (1/2)* sum(f.*ones(size(t))) * dx * 1/L;
    fFS = A0;
    for k = 1:N
        Ak(k) = sum(f.*cos(2*pi*k*t/L))*dx*1/L;
        Bk(k) = sum(f.*sin(2*pi*k*t/L))*dx*1/L;
        fFS = fFS + Ak(k)*cos(2*k*pi*t/L) + Bk(k)*sin(2*k*pi*t/L);
    end
    hold on;
    plot(t,fFS, 'r--', 'linewidth', 2);
    drawnow;
    pause(0.1);
end
```