Lecture 4a: Time-Domain Analysis of Continuous-Time Systems with MATLAB

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Outline

- MATLAB Codes

Zero-Input Response $y_0(t)$

To solve a differential equation, you can use a command dsolve to solve the equation. For an continuous-time LTI system specified by the differential equation

$$(D^2 + 4D + k)y(t) = (3D + 5)f(t)$$

determine the zero-input component of the response if the initial conditions are $y_0(0) = 3$, and $\dot{y}_0(0) = -7$ for two values of k: (a) 3 (b) 4 (c) 40.

```
(a) y0 = dsolve('D2y+4*Dy+3*y=0','y(0)=3','Dy(0)=-7','t')
    y0 = exp(-t) + 2*exp(-3*t)

(b) y0 = dsolve('D2y+4*Dy+4*y=0','y(0)=3','Dy(0)=-7','t')
    y0 = 3*exp(-2*t) - t*exp(-2*t)

(c) y0 = dsolve('D2y+4*Dy+40*y=0','y(0)=3','Dy(0)=-7','t')
    y0 = 3*cos(6*t)exp(-2*t) - (sin(6*t)*exp(-2*t))/6
```

Zero-Input Response $y_0(t)$

To plot y_0 respect to t, we can use a command eval as follow:

```
t = -1:0.01:10;
y = eval(y0);
plot(t,y);
```

Zero-State Response $y_s(t)$

For an LTI system specified by the differential equation

$$(D^2 + 3D + 2)y(t) = Df(t)$$

To calculate the zero-state response, we can use MATLAB to calculate as follow:

$$h(t) = b_n\delta + P(D)y_n(t)u(t)$$

In this case $b_n = 0$ and the initial values of $y_n(t)$ are $y_n(0^-) = 0$ and $\dot{y}(0^-) = 1$.

```
y_n = dsolve('D2y+3*Dy+2*y=0','y(0)=0','Dy(0)=1','t');
y_n = exp(-t) - exp(-2*t)
Dy_n = diff(y_n)
Dy_n = -exp(-t) + 2*exp(-2*t)
```

Therefore

$$h(t) = 0 + (2e^{-2t} - e^{-t})u(t)$$

Zero-State Response $y_s(t)$

If $f(t) = 10e^{-3t}$ we have

```
syms t tau
yn = dsolve('D2y+3*Dy+2*y=0','y(0)=0','Dy(0)=1','t');
Dyn = diff(yn)
ft = 10*exp(-3*t);

% convolution of f(t) and y_s(t)
ys = int(subs(ft,tau)*subs(Dyn,t-tau),tau,0,t);
ys = -5*exp(-3*t)*(exp(2*t) - 4*exp(t) + 3)
```

The last line is

$$y_s(t) = -5e^{-t} + 20e^{-2t} - 15e^{-3t},\ t \geq 0$$

Finally, the total response is $y(t) = y_0(t) + y_s(t)$.

Classical Method

Solve the differential equation

$$(D^2 + 3D + 2)y(t) = Df(t)$$

for the input $f(t) = 5t + 3$ if $y(0^+) = 2$ and $\dot{y}(0^+) = 3$.

```
syms t tau

f = '5*t+3';
Df = diff(f,t);
argx = strcat('D2y+3*Dy+2*y=',char(Df));
y = dsolve(argx,'y(0)=2','Dy(0)=3','t');
y = 2*exp(-t) - 5/2*exp(-2*t) + 5/2
```

The last line is

$$y(t) = 2e^{-t} - \frac{5}{2}e^{-2t} + \frac{5}{2}$$

Reference

1. Xie, W.-C., *Differential Equations for Engineers*, Cambridge University Press, 2010.
2. Goodwine, B., *Engineering Differential Equations: Theory and Applications*, Springer, 2011.
3. Kreyszig, E., *Advanced Engineering Mathematics*, 9th edition, John Wiley & Sons, Inc., 1999.
4. Lathi, B. P., *Signal Processing & Linear Systems*, Berkeley-Cambridge Press, 1998.