Lecture 3: Systems

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Outline

- Notation and meaning
- Some examples
- Classification of systems
- Block diagrams

Systems

Definition:

- A system transforms **input signals** into **output signals** (or **response**)
- A system is a function mapping input signals into output signals.

We concentrate on systems with one input and one output signal, i.e., **single-input, single-output (SISO)** systems.

notation:

- $y = Su$ or $y = S(u)$ means the system S acts on input signal u to produce output signal y
- $y = Su$ does not, in general, mean multiplication.

RC circuit

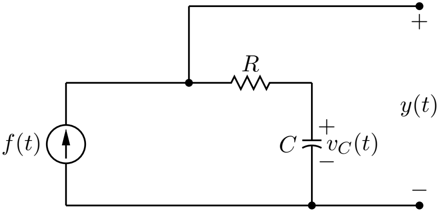


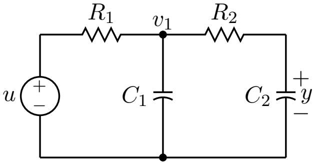
Figure: An example of a simple electrical system

The output voltage $y(t)$ is given by

$$y(t) = Rf(t) + \frac{1}{C}\int_{-\infty}^{t} f(\tau)d\tau$$

System examples

second-order RC circuit

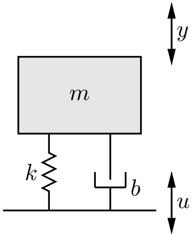


- current into C_2 is $C_2 \dfrac{dy}{dt} = \dfrac{v_1 - y}{R_2}$
- current input C_1 is $C_1 \dfrac{dv_1}{dt} = \dfrac{u - v_1}{R_1} - \dfrac{v_1 - y}{R_2}$
- using $v_1 = y(t) + R_2 C_2 \dfrac{dy}{dt}$ in the 2nd equation yields:

$$(R_1 C_1 R_2 C_2)\frac{d^2 y}{dt^2} + (R_1 C_1 + R_1 C_2 + R_2 C_2)\frac{dy}{dt} + y = u$$

System examples

Mass-Spring-Damper



(can represent suspension system, building during earthquake,...)

- $u(t)$ is displacement of base; $y(t)$ is displacement of mass
- spring force is $k(u-y)$; damping force is $b\frac{d}{dt}(u-y)$
- Newton's equation is $m\frac{d^2y}{dt^2} = b\frac{d}{dt}(u-y) + k(u-y)$ or

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = b\frac{du}{dt} + ku$$

System examples

a discrete-time system

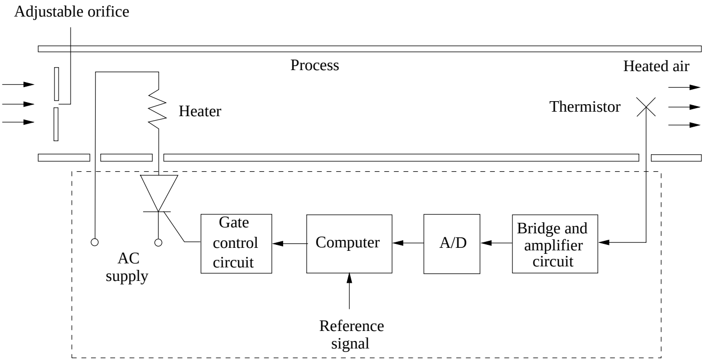


Figure: Discrete-time temperature control in an air-flow system

System examples

a discrete-time system

- F_n is the population of a predator (foxes), and R_n is the population of its prey (rabbits), where a and b are adapting parameters.
- r is the growth rate of the rabbit and c is the death rate of the fox.
- F_0 and R_0 are the initial population of each animal

$$R_n = rR_{n-1}(1 - R_{n-1}) - aR_{n-1}F_{n-1}$$

$$F_n = F_{n-1} + bR_{n-1}F_{n-1} - cF_{n-1}$$

Classification of Systems

- Linear and Nonlinear systems
- Constant-parameter and time-varying-parameter systems
- Instantaneous (memoryless) and dynamic (with memory) systems;
- Causal and noncausal systems
- Lumped-parameter and distributed-parameter systems
- Continuous-time and discrete-time systems
- Analog and Digital systems

a system F is **linear** if the following two properties hold:

1. **homogeneity:** if u is any signal and a is any scalar,

$$F(au) = aF(u)$$

2. **superposition:** if u_1 and u_2 are any two signals,

$$F(u_1 + u_2) = Fu_1 + Fu_2$$

in words, linearity means:

- scaling before or after the system is the same.
- summing before or after the system is the same.

A **nonlinear system** is a system which is not satisfied the homogeneity and superposition.

Classification of Systems

Time-Invariant and Time-Varying Parameter Systems

- Systems whose parameters do not change with time are **time-invariant** systems
- If the coefficients of the system are functions of time, then the system is a linear **time-varying** system.

$$L\frac{di(t)}{dt} + R(t)i(t) = f(t)$$

$R(t)$ is a coefficient of above system and it is a function of time, then the system is a linear time-varying system.

Continuous-Time Linear Time-Invariant Systems

example

Show that the system described by the equation

$$\frac{dy}{dt} + 3y(t) = f(t)$$

is linear

Let the system response to the inputs $f_1(t)$ and $f_2(t)$ by $y_1(t)$ and $y_2(t)$, respectively. Then

$$\frac{dy_1}{dt} + 3y_1(t) = f_1(t)$$
$$\frac{dy_2}{dt} + 3y_2(t) = f_2(t)$$

Multiplying the first equation by k_1, the second with k_2, and adding them yields

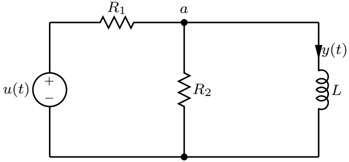
$$\frac{d}{dt}[k_1 y_1(t) + k_2 y_2(t)] + 3[k_1 y_1(t) + k_2 y_2(t)] = k_1 f_1(t) + k_2 f_2(t)$$

Therefor, when the input is $k_1 f_1(t) + k_2 f_2(t)$, the system response is $k_1 y_1(t) + k_2 y_2(t)$. Consequently, the system is linear.

From the previous example, we can readily generalize the result to show that a system described by a differential equation of the form

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_0 y = b_m \frac{d^m u}{dt^m} + \cdots + b_1 \frac{du}{dt} + b_0 u$$

is a linear system. The coefficients a_i and b_i in this equation can be constants or functions of time.



By using the Kirchoff's current law at node a

$$\frac{v_a(t)-u(t)}{R_1}+\frac{v_a(t)}{R_2}+y(t)=0$$

$$v_a(t)=L\frac{dy(t)}{dt}$$

we have

$$\frac{dy(t)}{dt}+\frac{R_1R_2}{L(R_1+R_2)}y(t)=\frac{R_2}{L(R_1+R_2)}u(t)$$

The last equation is in the linear differential equation form. Then, the system is linear. Moreover, since all coefficients of the equation are constant, the system is time-invariant.

- a **causal** (also known as a **physical** or **non-anticipative**) system is a system which the output at any instant t_0 depends only on the value of the input $u(t)$ for $t \leq t_0$.
- in other words, the value of the output at the present instant depends only on the past and present values of the input $u(t)$
- a **noncausal** (or **anticipative**) system is a system that violates the condition of causality.

For example if we apply an input starting at $t = 0$ to a noncausal system, the output would begin even before $t = 0$. For example

$$y(t) = f(t-2) + f(t+2)$$

Classification of Systems

Lumped-Parameter and Distributed-Parameter Systems

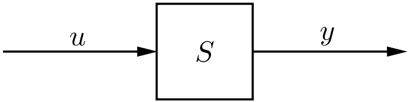
- a **lumped-parameter** system is a system where each component of the system is regarded as being lumped at one point in space. therefore, in lumped-parameter models, signals can be assumed to be functions of time t alone. For example, electronic circuits , etc. the system equations require only one independent variable and therefore are ordinary differential equations.
- a **distributed-parameter** system is a system where the system dimensions cannot be assumed to be small compared to the wavelengths of the signals such as transmission lines, waveguides, antennas, and microwave tubes, etc. The signals of this system are functions of space as well as of time, leading to mathematical models consisting of partial differential equations.

Classification of Systems

- **Continuous-time system** is a system whose inputs and outputs are continuous-time signals.
- **Discrete-time system** is a system whose inputs and outputs are discrete-time signals.
- If a continuous-time signal is sampled, the resulting signal is a discrete-time signal. We can process a continuous-time signal by processing its samples with a discrete-time system.
- **Analog system** is a system whose inputs and outputs are analog.
- **Digital system** is a system whose inputs and outputs are digital.

Block diagrams

systems often denoted by **block diagram**:



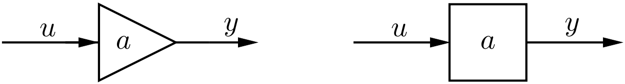
- lines with arrow denote signals
- boxes denotes systems: arrows show inputs and outputs
- special symbols for some systems

Block diagrams

scaling system: $y(t) = au(t)$

- called an *amplifier* if $|a| > 1$
- called an *attenuator* if $|a| < 1$
- called *inverting* if $a < 0$
- a is called the *gain* or *scaling factor*

Usually, denoted by triangle or rectangle in block diagram:

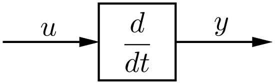


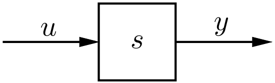
Block diagrams

Continuous-time system

differentiator: $y(t) = \dfrac{du}{dt}$

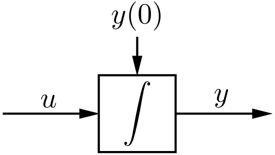
commonly used notations for differentiator:

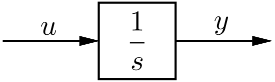




integrator: $y(t) = \int_a^t u(\tau)d\tau$ (a is often 0 or $-\infty$)

commonly used notations for integrator:

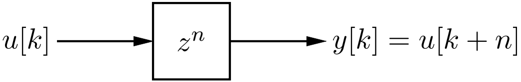




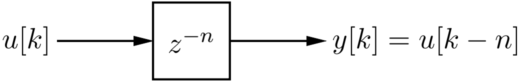
Block diagrams

Discrete-time system

forward-shift: $y[k] = z^n u[k] = u[k+n]$

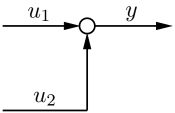


backward-shift: $y[k] = z^{-n} u[k] = u[k-n]$

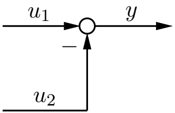


Example with multiple inputs

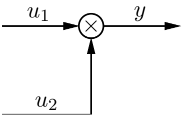
- **summing system:** $y(t) = u_1(t) + u_2(t)$



- **difference system:** $y(t) = u_1(t) - u_2(t)$



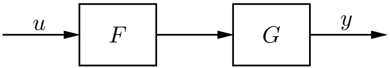
- **multiplier system:** $y(t) = u_1(t)u_2(t)$



Interconnections of systems

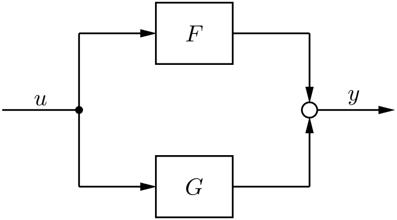
we can interconnect systems to form new systems, e.g.,

- **cascade (or series):** $y = G(Fu) = GFu$



(note the block diagrams and algebra are reversed)

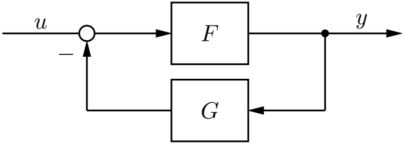
- **sum (or parralled):** $y = Fu + Gu$



Interconnections of systems

cont.

- **Feedback:** $y = F(u - Gy)$

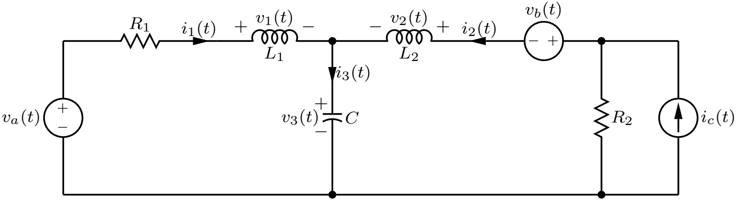


  - the minus sign is for a negative feedback while the plus sign is for a positive feedback.

Block Diagram

example

A circuit shown below has three state equations:



$$v_1(t) = v_a(t) - R_1 i_1(t) - v_3(t)$$

$$v_3(t) = -v_2(t) - v_b(t) + R_2(i_c(t) - i_2(t))$$

$$v_2(t) = -R_2 i_2(t) - v_3(t) - v_b(t) + R_2 i_c(t)$$

$$i_3(t) = i_1(t) + i_2(t)$$

Block Diagram
example

Since

$$v_1(t) = L_1 \frac{di_1(t)}{dt},$$
$$v_2(t) = L_2 \frac{di_2(t)}{dt},$$
$$i_3(t) = C \frac{dv_3(t)}{dt},$$

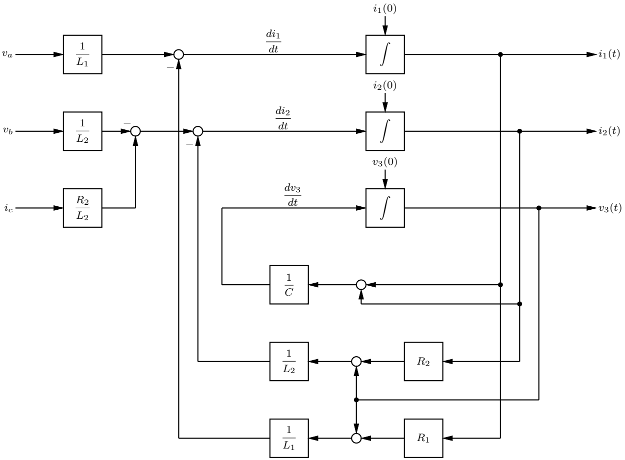
we obtain the dynamic equation in state form :

$$\frac{di_1(t)}{dt} = -\frac{R_1}{L_1} i_1(t) - \frac{1}{L_1} v_3(t) + \frac{1}{L_1} v_a(t) \quad (1)$$
$$\frac{di_2(t)}{dt} = -\frac{R_2}{L_2} i_2(t) - \frac{1}{L_2} v_3(t) - \frac{1}{L_2} v_b(t) + \frac{R_2}{L_2} i_c(t) \quad (2)$$
$$\frac{dv_3(t)}{dt} = \frac{1}{C} i_1(t) + \frac{1}{C} i_2(t) \quad (3)$$

Block Diagram

example

Note that our required outputs are $i_1(t), i_2(t)$, and $v_3(t)$.



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